How AI was used in

2-large sets are sets of nil-Bohr recurrence

A report on the work of Ryan Alweiss and Sohail Farhangi

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AI contributed substantially to the mathematics, exposition, and checking of this paper. Its role evolved during roughly fourteen months of work: initially it supported an author-led attempt to extend Alweiss's random-coloring method from tori to nilmanifolds; later it generated a different, simpler proof and helped develop stronger results and applications. The current preprint uses this later *shell-coloring* approach. Most of the long video concerns the earlier approach, so the volume of discussion should not be mistaken for the distribution of contributions to the final manuscript. [1] [2, 4:48]

1. Learning, developing, and testing the original approach. ChatGPT served as an interactive mathematical tutor, explaining nilmanifolds, Mal'cev coordinates, equidistribution, Lie theory, and differential geometry. Farhangi used it to unpack unfamiliar arguments, locate references, work through examples, and turn informal ideas into precise lemmas. The authors supplied the research problem, proof strategy, quantitative constraints, and objections. Uploaded TeX drafts dated June 23, June 27, and July 2, recovered from the data export, document an evolving manuscript and author-identified gaps before the later AI completions; they are not evidence of an exclusively human-written stage.

AI also proposed proof ingredients and exposed obstructions. A notable Heisenberg example revealed a base-point problem in the attempted argument; AI then suggested an enlarged nilsystem that retained the missing information. Farhangi pursued concrete examples and demanded a rigorous general construction. Such exchanges could materially redirect the research. They also produced dead ends: some references did not contain the claimed statements, apparently convincing arguments failed when written out, and an extended investigation of orbit covering missed a shorter route through existing Green–Tao theory. [2, 1:47:53] [2, 2:28:31]

2. A new proof, stronger conclusions, and applications. Once the original draft was substantially developed, the authors asked GPT-6/Astra to complete its remaining technical details. Alweiss then asked ChatGPT whether the result could be proved through direct calculations with special generalized polynomials, avoiding the heavy nilmanifold machinery. This request produced the shell-coloring proof underlying the current preprint. Farhangi asked for a longer, more explicit version of the short AI-generated proof and studied it through repeated questions about individual steps. This was substantive proof generation followed by human-directed development, not merely language editing. [1] [2, 3:00:23]

AI suggested quantitative refinements involving shrinking approximation targets. Farhangi posed a fixed-density analogue, and AI developed a polychromatic statement controlling progressions that omit a color, from which both partition and density consequences follow. The authors shaped the formulation and chose how to balance technical strength with readable presentation. When asked for applications, AI proposed a comparison of Szemerédi extremal numbers with and without a nil-Bohr restriction on the common difference. Farhangi explicitly asked to improve the initial factor-two comparison to $1+\varepsilon$; AI supplied the strengthened argument. This application helped establish the value of retaining the quantitative result. The record therefore supports both substantial AI input and specific human choices that sharpened the outcome. [1] [2, 3:12:53]

3. Drafting, explanation, and refinement. AI produced complete intermediate TeX documents, expanded compressed proofs, combined results into a draft, and helped revise outlines, notation, matrix calculations, and explanatory prose. Farhangi repeatedly asked why assumptions were needed and requested expansions or shorter replacements until he could understand and present the argument. Alweiss personally rewrote the simpler Bohr-case exposition. AI also helped identify the precise finite-partition averaging property used in the density deduction and a counterexample showing that finite subadditivity alone was insufficient. Literature searches helped locate precedents for the polychromatic principle; Farhangi requested pinpoint references and opened the cited papers to check them. Exploration of wider groups, distal systems, related examples, and further conjectures informed the authors' choices, but many proposed extensions were left out or remained unresolved. [2, 3:26:25] [2, 3:28:55]

4. Checking and human responsibility. The workflow included repeated requests to audit proofs and drafts for mathematical errors, missing hypotheses, unclear exposition, and typos. These checks found issues in both AI-generated and human-written material, including an incorrect constant. They were not accepted automatically: Farhangi challenged alleged errors, rejected some proposed fixes, and requested further explanations and revisions. Conversely, AI sometimes gave wrong examples, misplaced confidence, or proofs that lost an error term. The authors distinguished exploratory output from material ready for publication: some side investigations were not read in full and were not incorporated as established results. Human responsibility lay in choosing the questions, understanding and checking the mathematics, verifying references, deciding what to include, and accepting responsibility for the final paper. [2, 3:39:13] [2, 3:46:55]

5. Formalization and the scope of this record. Codex also assisted with a Lean formalization. The project's coverage records report formal checks of the arithmetic main results and the Szemerédi-number consequence, while explicitly excluding the nilmanifold characterization used in Section 4. In the video, Farhangi stated that he had not yet checked the certificate himself. The recorded formalization work should therefore be distinguished from a claim that every statement and interpretation in the paper had been independently certified. [2, 2:58:55]

The public materials provide unusually detailed evidence of this collaboration, but not a complete allocation of intellectual credit: some chats were deleted, Alweiss's parallel interactions are only partly represented, and uploaded drafts can already contain earlier AI assistance. The central lesson of this particular record is that AI supplied explanations, candidate arguments, a new proof route, and useful applications, while the authors supplied sustained mathematical direction, scrutiny, selection, and exposition.

Sources. [1] Alweiss–Farhangi, 2-*large sets are sets of nil-Bohr recurrence*, arXiv:2610.16605v2, especially the AI statement. [2] Farhangi's recorded walkthrough; linked timestamps above locate representative passages. [3] Supplementary-materials page, including the surviving chat logs and intermediate AI outputs. The uploaded draft attachments and the local formalization coverage record were also consulted. This is a concise account of documented use, not a new mathematical verification.