Problem 1: Suppose $y = f(x)$ is a continuous and positive function on $[a, b]$. Let $\mathcal{S}$ be the surface generated when the graph of $f(x)$ is revolved about the x-axis.

(a) Show that $\mathcal{S}$ is described parametrically by $\vec{r}(u, v) = \langle u, f(u)\cos(v), f(u)\sin(v)\rangle$, for $a \leq u \leq b$, $0 \leq v \leq 2\pi$.

(b) Find an integral that gives ths surface area of $\mathcal{S}$.

(c) Apply the result of part (b) to the surface $\mathcal{S}_1$ generated with $f(x) = x^3$, for $1 \leq x \leq 2$.

Problem 2: Given a sphere of radius R and a length $0 < L \leq 2R$, show that the surface area of the strips of length L on the sphere depend only on L and not on the location of the strip.

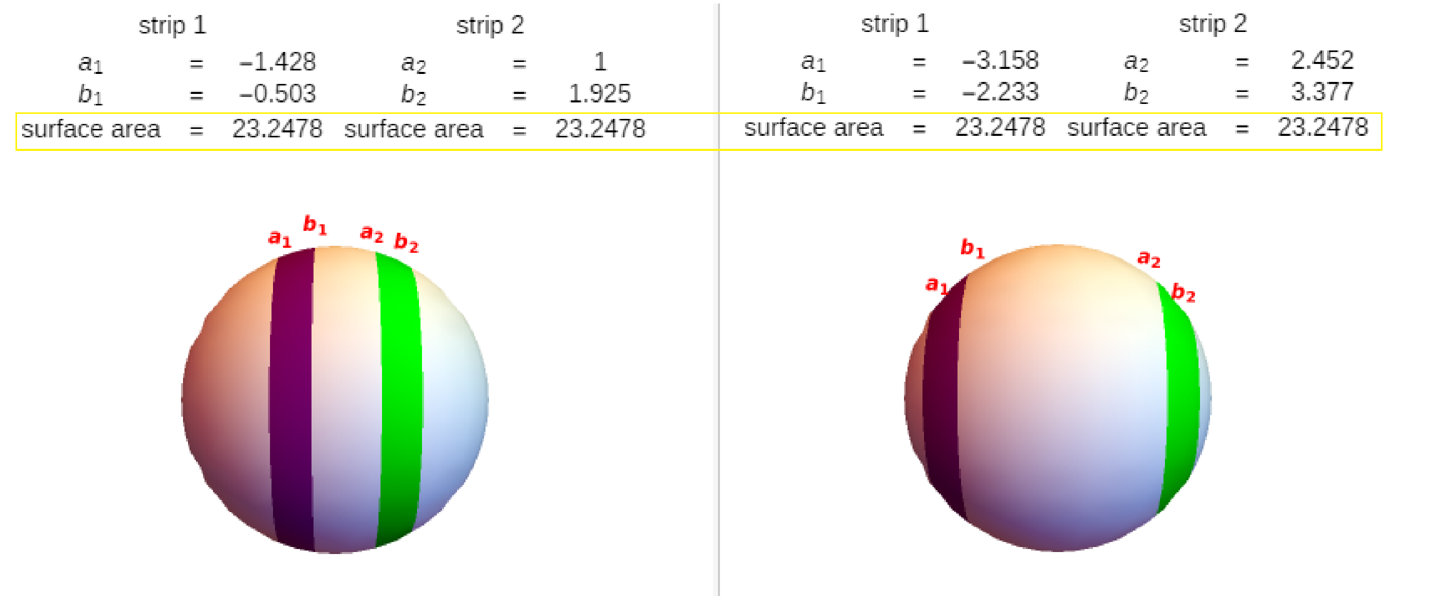


Figure 1: An example of Problem 2 with $L = 0.925$ and $R = 4$.

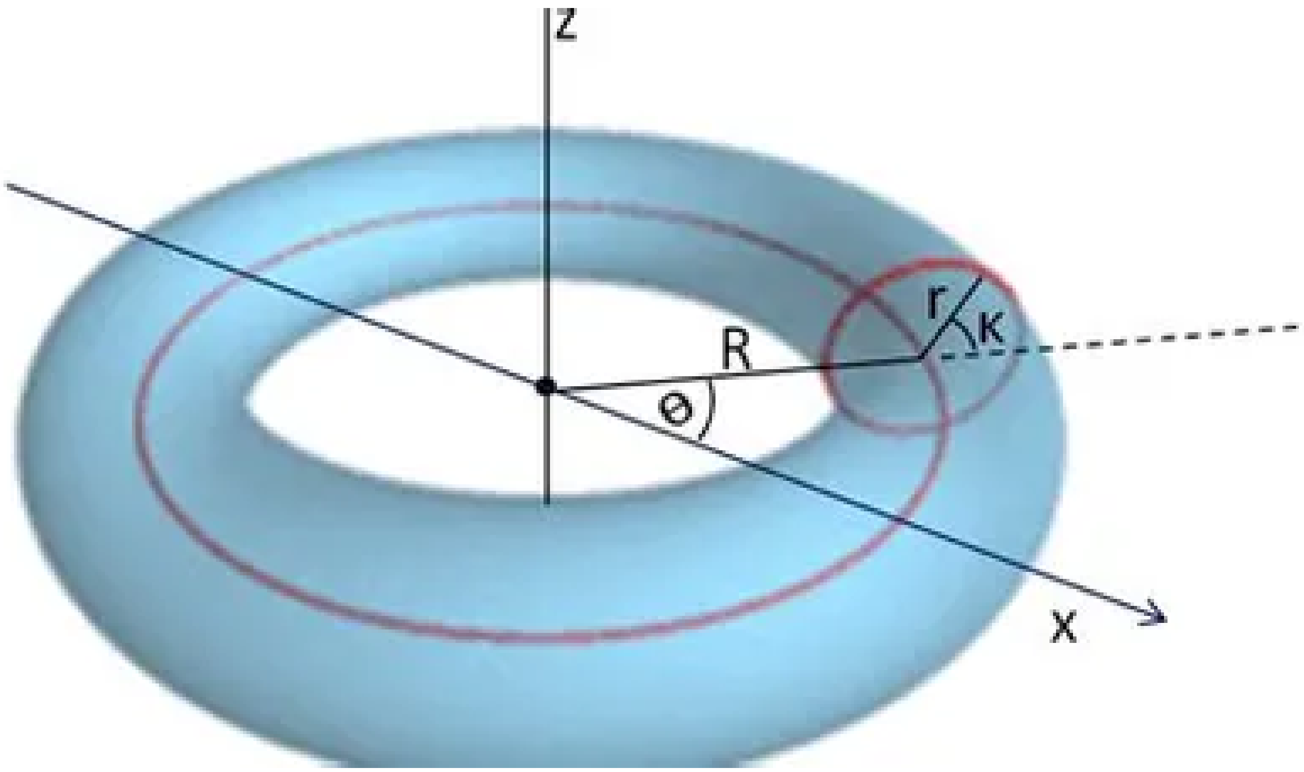
Hint: Problem 1 can help.

Problem 3(Surface Area and Volume of a Torus)**:**

(a) Show that a torus T with radii $R > r$ (See figure) may be described parametrically by $r(K, \theta) = \langle (R + r\cos(K))\cos(\theta), (R + r\cos(K))\sin(\theta), r\sin(K)\rangle$, for $0 \leq K \leq 2\pi$, $0 \leq \theta \leq 2\pi$.

(b) Show that the surface area of the torus T is $4\pi^2 Rr$.
Interestingly, the arclength of the small circle is $2\pi r$ and the arclength of the large circle inside the torus is $2\pi R$, so the surface area of the torus happens to be the product of the arclengths of the 2 circles from which it is created.

(c) Use part (a) to find a parametrization $\vec{s}(K, \theta, r)$ for the solid torus $\mathcal{T}$ (T from part (a) as well as its interior), then use $\vec{s}$ and a change of variables to show that the volume of $\mathcal{T}$ is $\pi r^2 R$.



Problem 4: Let $z = s(x, y)$ define the surface $\mathcal{S}$ over a region R in the xy-plane, where $z \geq 0$ on R. Show that the downward flux of the vertical vector field $\vec{F} = \langle 0, 0, -1 \rangle$ across $\mathcal{S}$ equals the area of R. Interpret the result physically.

Problem 5: Let $\mathcal{S}$ be the upper half of the ellipsoid $\frac{x^2}{4}+\frac{y^2}{9}+z^2=1$ and let $\vec{F}=\langle z,x,y\rangle$. Use Stoke's theorem to evaluate

$$\iint_{\mathcal{S}}(\nabla\times\vec{F})\cdot\hat{n}dS. \tag{1}$$

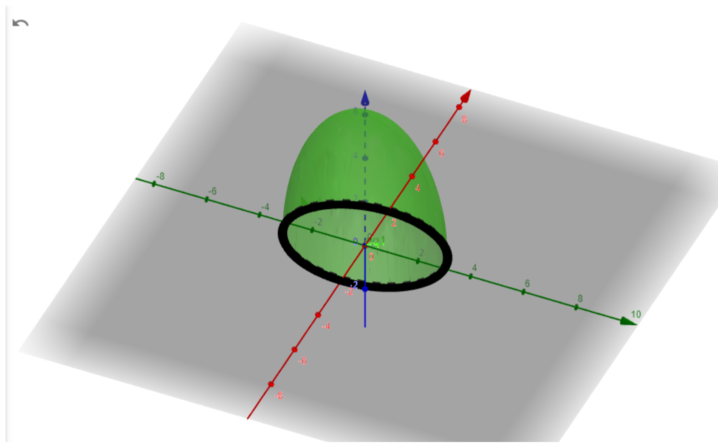


Figure 2: A view of $\mathcal{S}$ and $\partial\mathcal{S}$.

Problem 6: Let C be the circle $x^2 + y^2 = 12$ in the plane $z = 0$ (as a subset of $\mathbb{R}^3$) and let $\vec{F} = \langle (x+4)^x, y\ln(y+4), e^{z^2+\sqrt{z}}\rangle$. Use Stoke's theorem to evaluate

$$\oint_C \vec{F} \cdot d\vec{r}. \tag{2}$$

Problem 7: Let $\mathcal{S}$ be the surface of the cube cut from the first octant by the planes $x = 1, y = 1$, and $z = 1$. Let $\vec{F} = \langle x^2, 2xz, y^2 \rangle$. Use the Divergence theorem to evaluate the net outward flux of $\vec{F}$ across $\mathcal{S}$.

Problem 8: Let $\mathcal{S}$ be the boundary of the ellipsoid $\frac{x^2}{4} + y^2 + z^2 = 1$ and let $\vec{F} = \langle x^2 e^y \cos(z), -4xe^y \cos(z), 2xe^y \sin(z)\rangle$. Evaluate the outward flux of $\vec{F}$ across $\mathcal{S}$.
