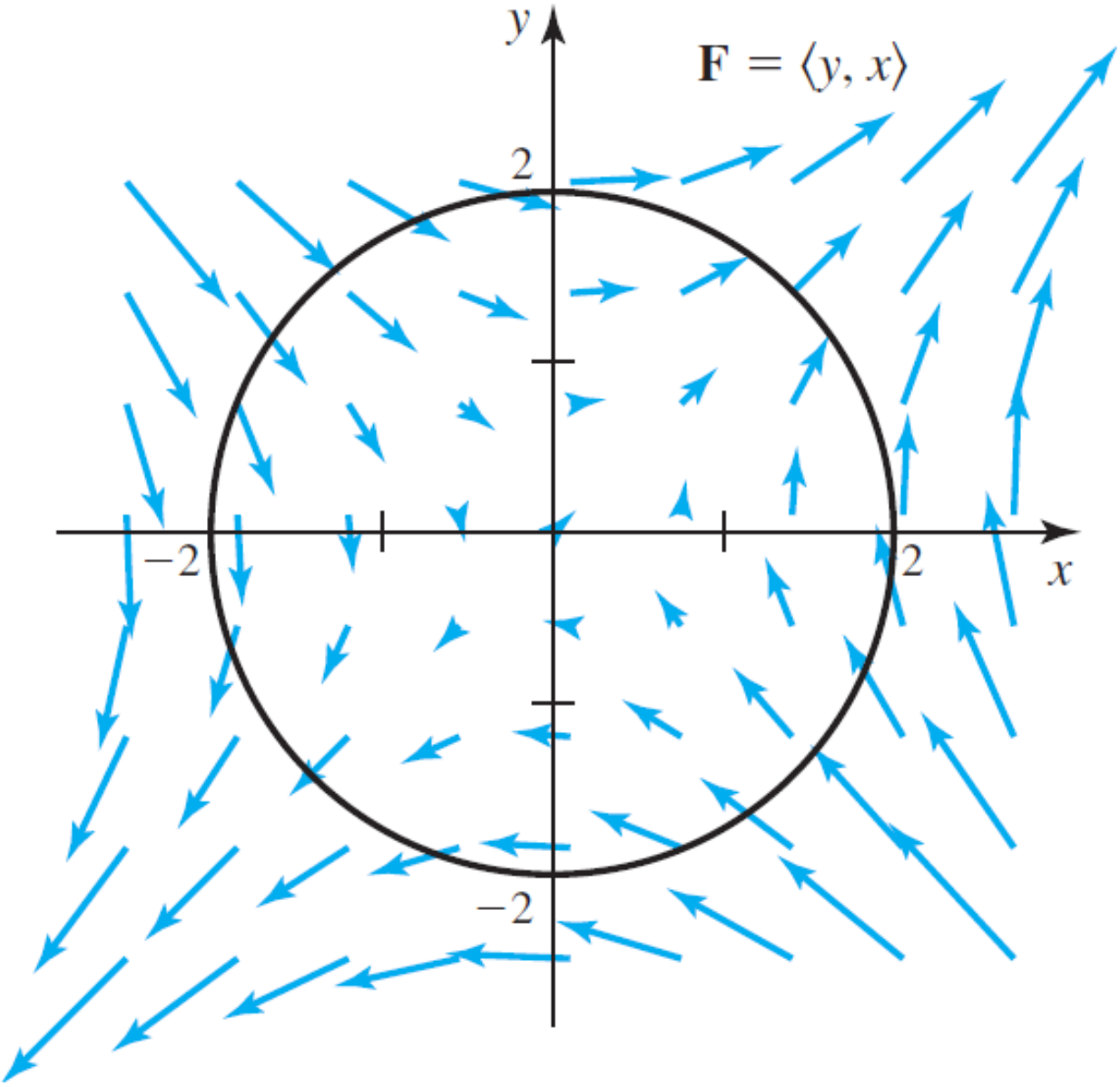
Problem 1(Repeated from last week)**:** Consider the flow field $\mathbf{F} = \langle y, x \rangle$ shown in the figure below.



(a) Compute the outward flux across the quarter circle C:$\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t) \rangle$, $0 \leq t \leq \frac{\pi}{2}$.

(b) Compute the outward flux across the quarter circle C:$\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t) \rangle$, $\frac{\pi}{2} \leq t \leq \pi$.

(c) Explain why the flux across the quarter circle in the third quadrant equals the flux computed in part **a**.

(d) Explain why the flux across the quarter circle in the fourth quadrant equals the flux computed in part **b**.

(e) What is the outward flux across the full circle?

Problem 2: An idealized two-dimensional ocean is modeled by the square region $R = [-\frac{\pi}{2}, \frac{\pi}{2}] \times [-\frac{\pi}{2}, \frac{\pi}{2}]$. with boundary $\mathcal{C}$. Consider the stream function $\Psi(x, y) = 4\cos(x)\cos(y)$ defined on R. Some of the level curves of Ψ are shown in the figure below.

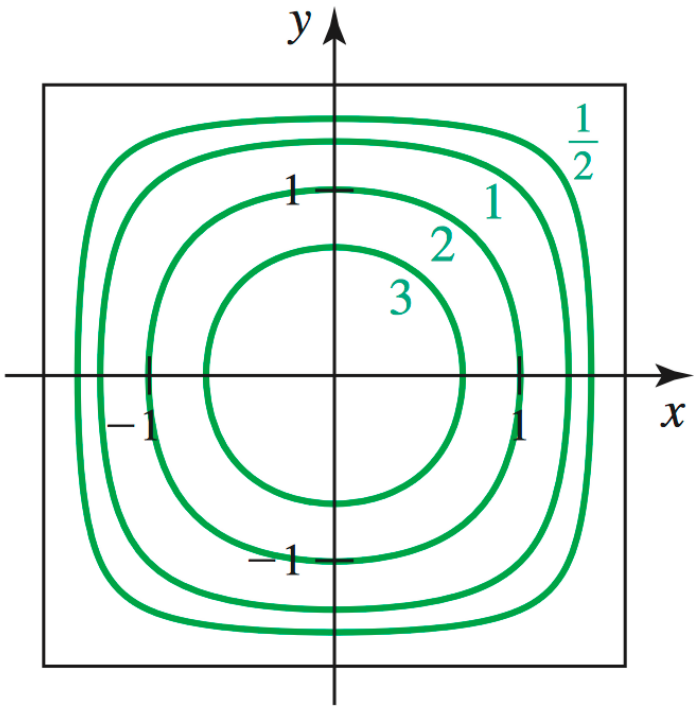


Figure 1: Some level curves of the stream function $\Psi(x, y)$.

(a) The horizontal (east-west) component of the velocity is $u = \Psi_y$ and the vertical (north-south) component of the velocity is $v = -\Psi_x$. Sketch a few representative velocity vectors and show that the flow is counterclockwise around the region.

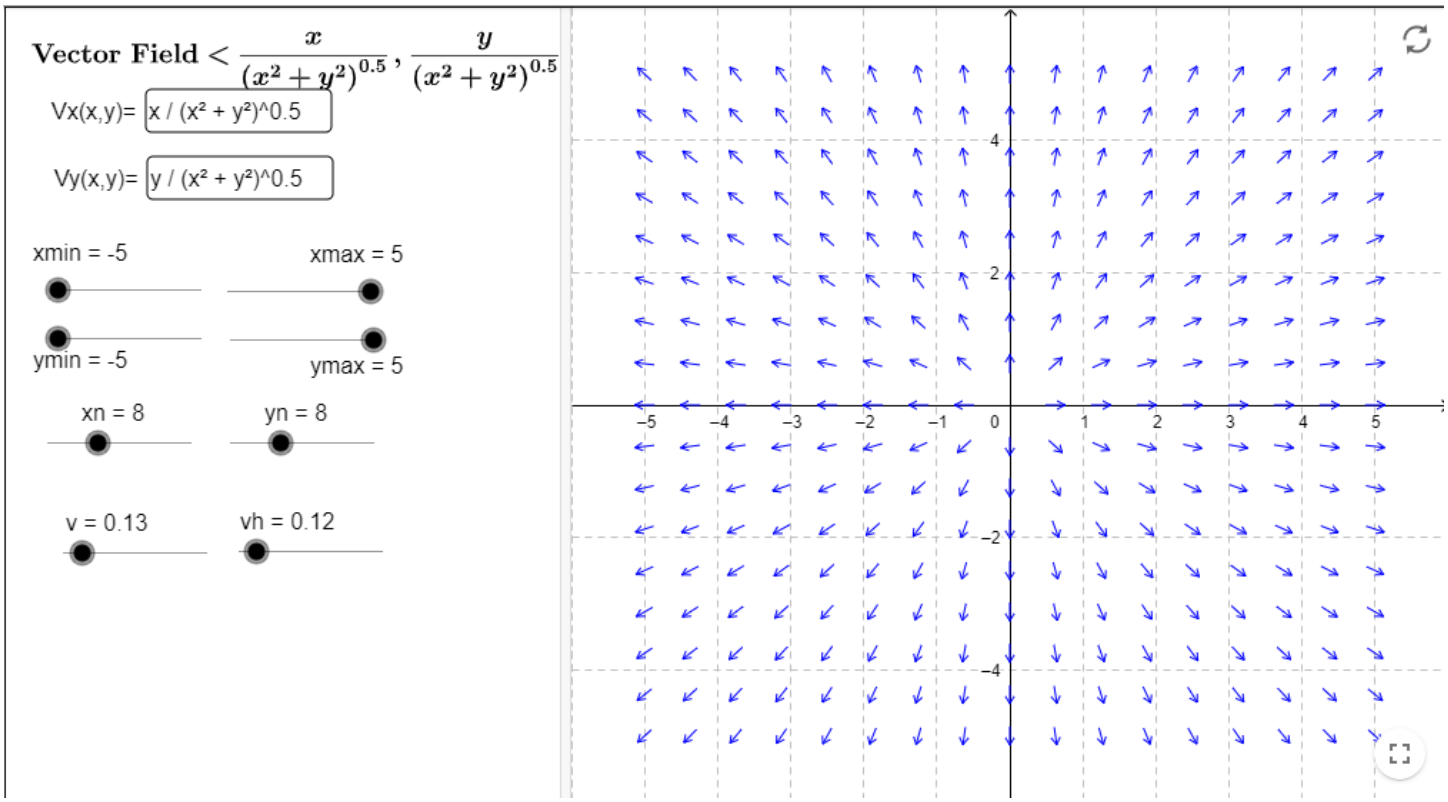
(b) Is the velocity field source free? Explain.

(c) Is the velocity field irrotational? Explain.

(d) Find the total outward flux across $\mathcal{C}$.

(e) Find the circulation on $\mathcal{C}$ assuming counterclockwise orientation.

Problem 3: Consider the radial field $\vec{F}(x,y) = \frac{\langle x,y\rangle}{\sqrt{x^2+y^2}} = \frac{\vec{r}}{|\vec{r}|}$ shown below.



(a) Explain why the conditions of Green's Theorem do not apply to $\vec{F}$ on a region R containing the origin.

(b) Let R be the unit disk centered at the origin and compute

$$\iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dA. \tag{1}$$

(c) Evaluate the line integral in the flux form of Green's Theorem applied to the region R and the vector field $\vec{F}$.

(d) Do the results of parts (b) and (c) agree? Explain.