Problem 1: Let R be the region bounded by the lines $y - x = 0, y - x = 2, y + x = 0, y + x = 2$. Use a change of variables to evaluate

$$\iint_R \sqrt{y^2 - x^2}\, dA. \tag{1}$$

Problem 2: Let R be the region in the first quadrant bounded by the hyperbolas $xy = 1$ and $xy = 4$ and the lines $y = x$ and $y = 3x$. Evaluate

$$\iint_R y^4 dA. \tag{2}$$

Note that you can also solve this problem in Cartesian coordinates and polar coordinates, not just a change of variables. Try solving it with all three methods and compare their difficulties!

Problem 3: Find the volume of the solid D that is bounded by the planes $y - 2x = 0, y - 2x = 1, z - 3y = 0, z - 3y = 1, z - 4x = 0$, and $z - 4x = 3$.

Problem 4: This problem has parts **a.-g.** spread out across the following pages. Your solutions to parts **a, b,** and **f** need (hand drawn or computer generated) pictures.

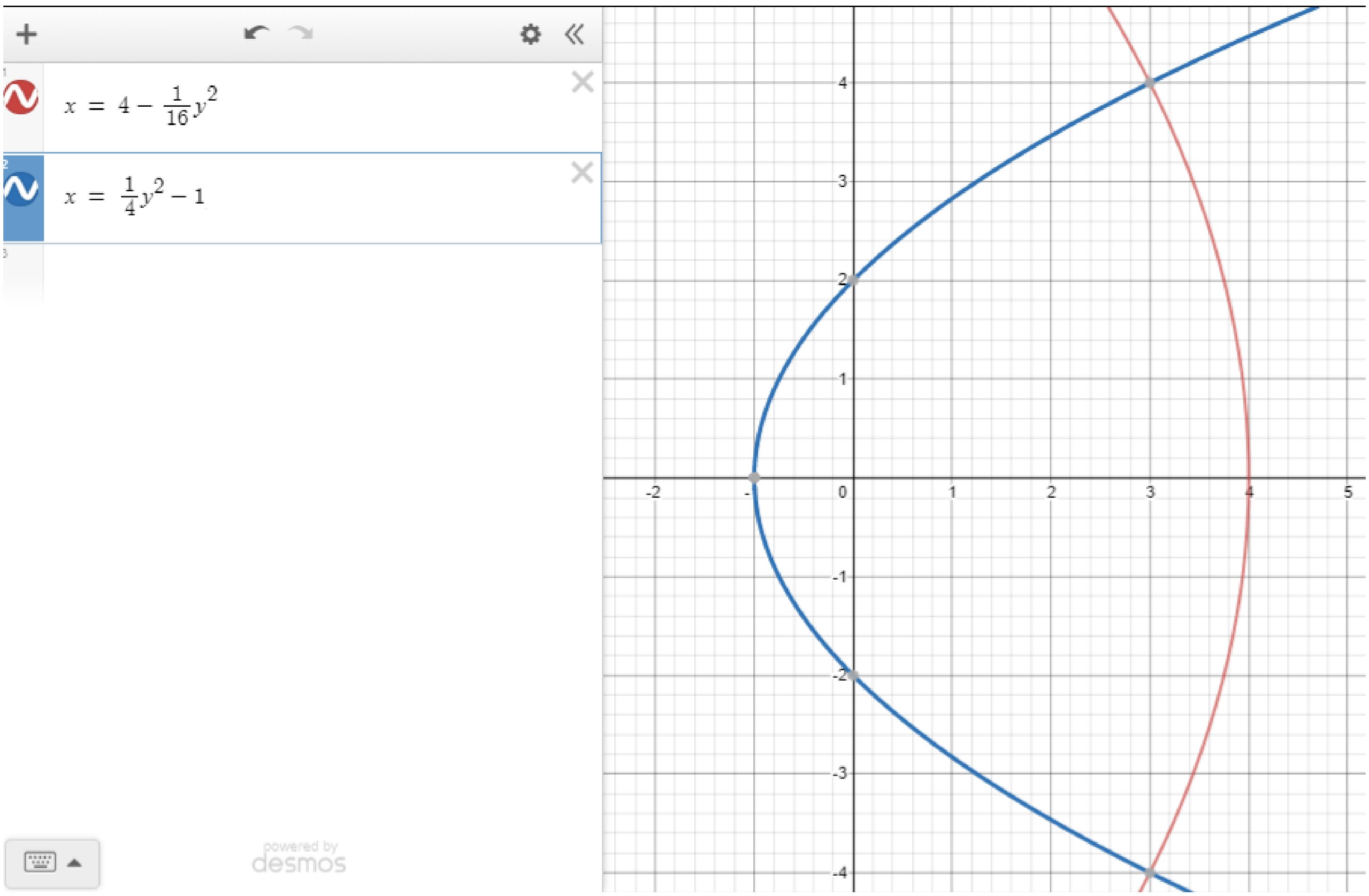
Consider the Transformation T from the uv-plane to the xy-plane given by $T(u,v) = (u^2 - v^2, 2uv)$.

a. Show that the lines $u = a$ in the uv-plane map to parabolas in the xy-plane that open in the negative x-direction with vertices[1] on the positive x-axis.[2] Compare the images of the lines $u = a$ and $u = -a$ under T.

b. Show that the lines $v = b$ in the uv-plane map to parabolas in the xy-plane that open in the positive x-direction with vertices on the negative x-axis.[3] Compare the images of the lines $v = b$ and $v = -b$ under T.

c. Evaluate $J(u,v)$.

d. Use a change of variables into parabolic coordinates to find the area of the region R in the xy-plane bounded by the curves $x = 4 - \frac{1}{16}y^2$ and $x = \frac{1}{4}y^2 - 1$. Sketch a picture of the new region of integration as well.



[1]The vertex of the parabola $y = x^2$ is the point $(0,0)$ and the vertex of the parabola $x = y^2$ is also $(0,0)$.

[2]You have to show that the curve $\vec{r_1}(v) = (a^2 - v^2, 2av)$ represents the same curve as $x - c = -by^2$ for some positive numbers b and c.

[3]You have to show that the curve $\vec{r_2}(u) = (u^2 - b^2, 2ub)$ represents the same curve as $x - c = by^2$ for some positive number b and some negative number c.

e. Use a change of variables into parabolic coordinates to find the area of the curved rectangle R above the x-axis bounded by $x = 4 - \frac{1}{16}y^2, x = 9 - \frac{1}{36}y^2, x = \frac{1}{4}y^2 - 1$, and $x = \frac{1}{64}y^2 - 16$. Sketch a picture of the new region of integration as well.

f. Describe the effect of the transformation $(u, v) \mapsto (2uv, u^2 - v^2)$ on horizontal and vertical lines in the uv-plane.[4]

g. Show that the parabolas that are the images of the lines $u = a$ and $v = b$ under $T(u, v) = (u^2 - v^2, 2uv)$ are orthogonal to eachother.

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[4]Remember that the transformation $(x, y) \mapsto (y, x)$ reflects points in the xy-plane across the line $y = x$. It will also help to use the results of parts **a.** and **b.** of this problem.