Problem 1: Suppose that the second partial derivative of f are continuous on $R = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$. Show that

$$\iint_R \frac{\partial^2 f}{\partial x \partial y}(x, y) dA = f(a, b) - f(a, 0) - f(0, b) + f(0, 0). \tag{1}$$

Hint: Think about the fundamental theorem of calculus.

Problem 2: Let $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

(a) Evaluate $\iint_R \cos(x\sqrt{y})dA$.

(b) Evaluate $\iint_R x^3 y \cos(x^2 y^2)dA$.

Hint: Choose a convenient order of integration.

Problem 3: Let R be the region inside of the ellipse $\frac{x^2}{18} + \frac{y^2}{36} = 1$ for which we also have $y \leq \frac{4}{3}x$.

(a) Find the area of R.

(b) Evaluate

$$\iint_R xy dA. \tag{2}$$

Problem 4: Let R be the region that is bounded by both branches of $y = \frac{1}{x}$, the line $y = x + \frac{3}{2}$, and the line $y = x - \frac{3}{2}$.

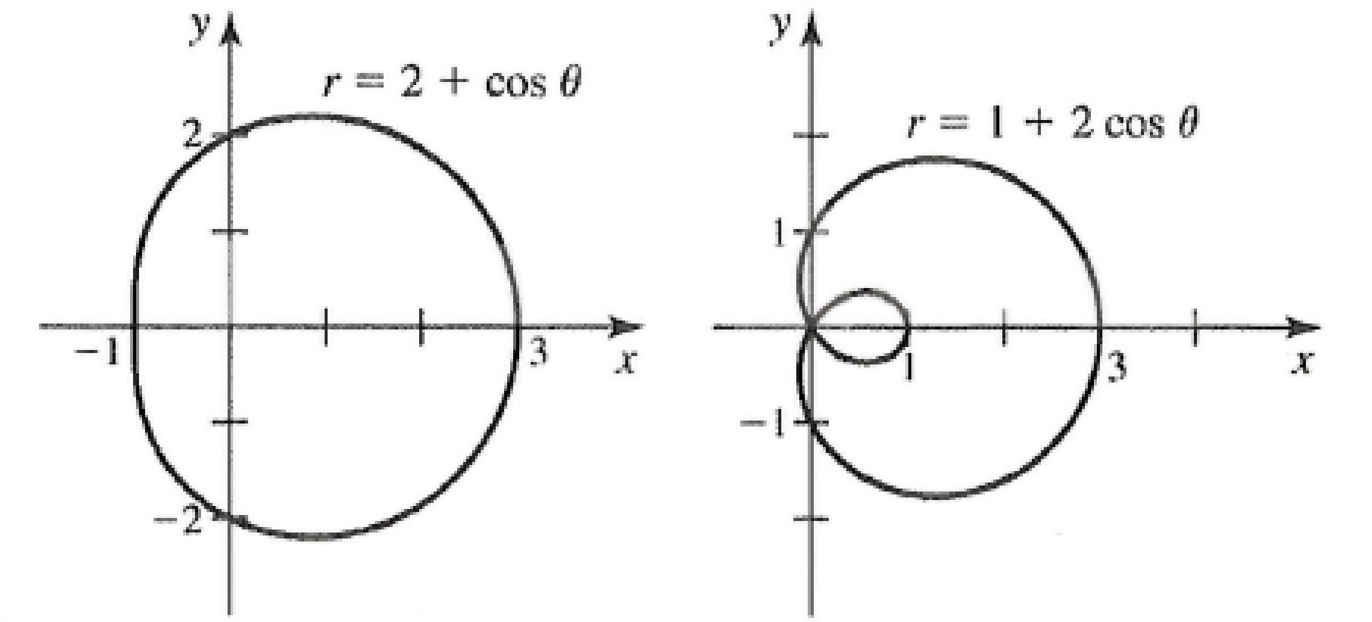
(a) Find the area of R.

(b) Evaluate

$$\iint_R xy dA. \tag{3}$$

Problem 5: Find the volume of the solid bounded by the planes $x = 0, x = 5, z = y - 1, z = -2y - 1, z = 0$, and $z = 2$.

Problem 6: The limaçon $r = b + a\cos(\theta)$ has an inner loop if $b < a$ and no inner loop if $b > a$.



(a) Find the area of the region bounded by the limaçon $r = 2 + \cos(\theta)$.

(b) Find the area of the region outside the inner loop and inside the outer loop of the limaçon $r = 1 + 2\cos(\theta)$.

(c) Find the area of the region inside the inner loop of the limaçon $r = 1 + 2\cos(\theta)$.

Problem 7: Let R be the region inside both the cardiod $r = 1 + \sin(\theta)$ and the cardiod $r = 1 + \cos(\theta)$. Sketch a picture of the region R, or create an image of the region R using a graphing program, then use double integration to find the area of R.

Problem 8: Find the volume of the solid S bounded by the paraboloid $z = 8 - x^2 - 3y^2$ and the hyperbolic paraboloid $z = x^2 - y^2$.

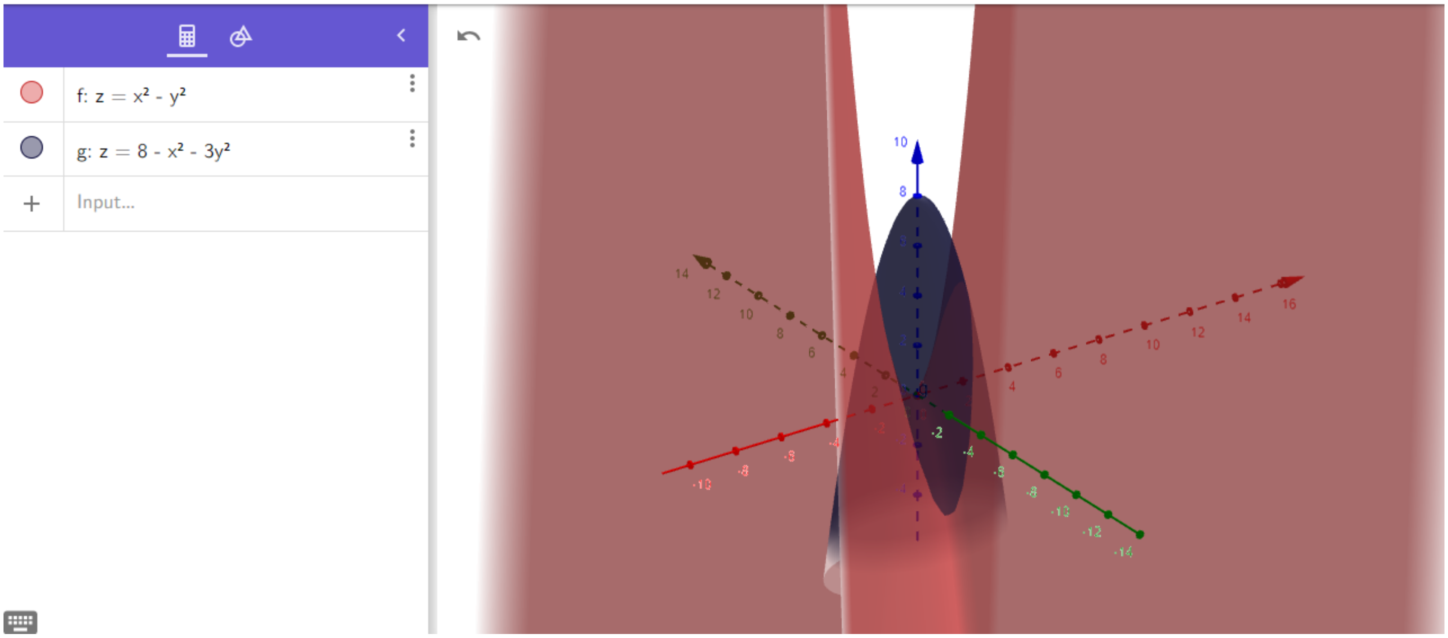


Figure 1: A view of the solid S whose volume we are calculating.