Problem 1: Verify that

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x)+\sin(y)}{x+y} = 1. \tag{1}$$

Problem 2: Consider the function

$$f(x,y) = \frac{xy^2}{x^2+y^4}. \tag{2}$$

(a) Show that if L is a line that passes through the origin, then

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in L}} f(x,y) = 0. \tag{3}$$

(b) Show that

$$\lim_{(x,y)\to(0,0)} f(x,y) \tag{4}$$

does not exist.

Problem 3: Consider the function $f(x, y) = \sqrt{|xy|}$.

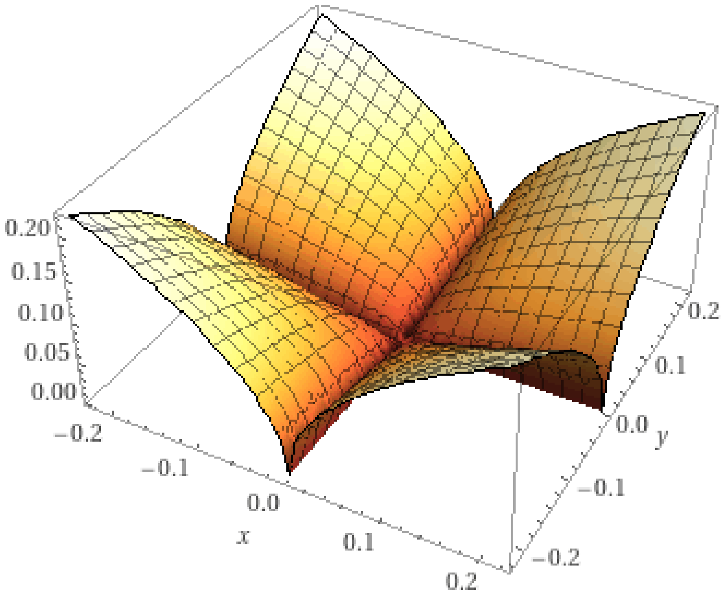

Figure 1: A graph of $z = \sqrt{|xy|}$.

(a) Is f continuous at $(0, 0)$?

(b) Show that $f_x(0, 0)$ and $f_y(0, 0)$ exist by calculating their values.

(c) Determine whether f_x and f_y are continuous at $(0, 0)$.

(d) Is f differentiable at $(0, 0)$?

Problem 4: Imagine a string that is fixed at both ends (for example, a guitar string). When plucked, the string forms a standing wave. The displacement u of the string varies with position x and with time t. Suppose it is given by $u = f(x,t) = 2\sin(\pi x)\sin(\frac{\pi}{2}t)$, for $0 \leq x \leq 1$ and $t \geq 0$ (see figure). At a fixed point in time, the string forms a wave on $[0,1]$. Alternatively, if you focus on a point on the string (fix a value of x), that point oscillates up and down in time.

(a) What is the period of the motion in time?

(b) Find the rate of change of the displacement with respect to time at a constant position (which is the vertical velocity of a point on the string).

(c) At a fixed time, what point on the string is moving fastest?

(d) At a fixed position on the string, when is the strong moving fastest?

(e) Find the rate of change of the displacement with respect to position at a constant time (which is the slope of the string).

(f) At a fixed time, where is the slope of the string greatest?

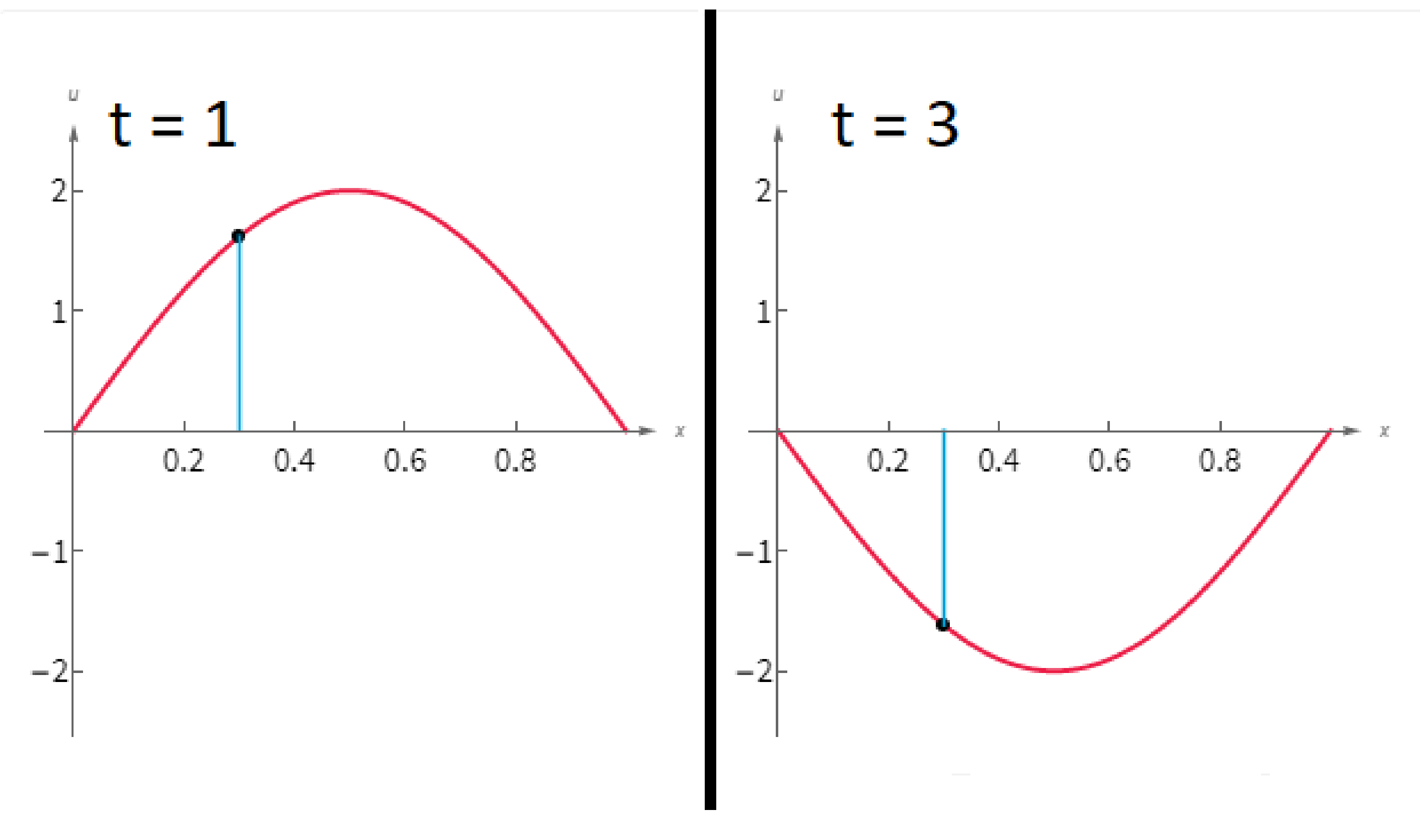

Figure 2: Snapshots of the wave at times $t = 1$ and $t = 3$.