Problem 1: Determine whether the following statements are true or false. If a statement is true, then explain why. If a statement is false, then provide a counterexample.

(a) The position, unit tangent, and principal unit normal vectors ($\vec{r}, \hat{T}$, and $\hat{N}$) at a point lie in the same plane.

(b) The vectors $\hat{T}$ and $\hat{N}$ at a point depend on the orientation of a curve.

(c) The curvature at a point depends on the orientation of a curve.

(d) An object with unit speed ($|\vec{v}| = 1$) on a circle of radius R has an acceleration of $\vec{a} = \frac{1}{R}\hat{N}$.

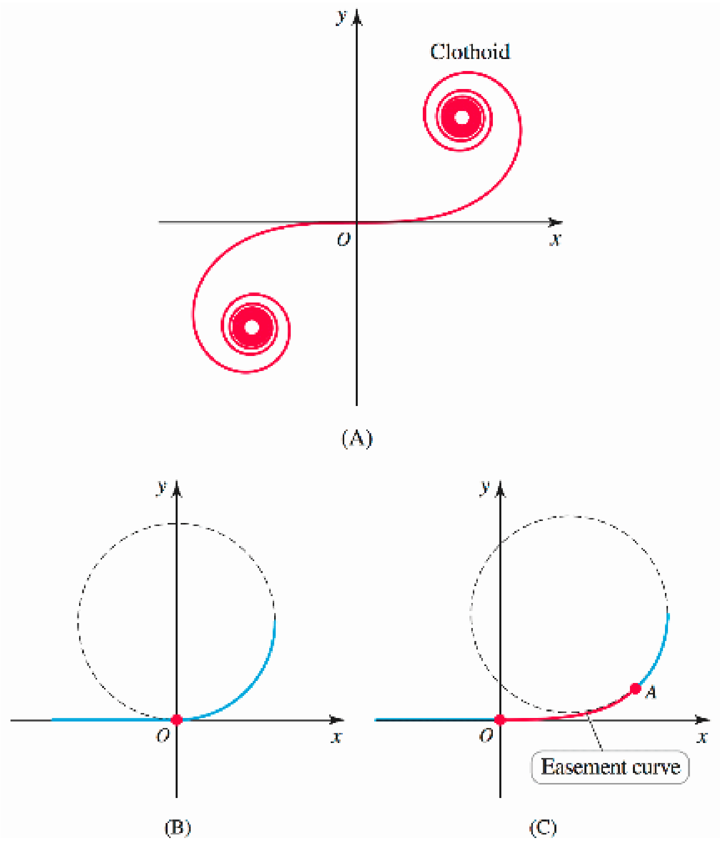
(e) If the speedometer of a car reads a constant 60 mi/hr, the car is not accelerating.

(f) A curve in the xy-plane that is concave up at all points has positive torsion.

(g) A curve with large curvature also has large torsion.

Problem 2: Compute the unit binormal vector $\hat{B}$ and torsion τ of the curve parameterized by $\vec{r}(t) = \langle 2\cos(t), 2\sin(t), -t\rangle, t \in \mathbb{R}(-\infty < t < \infty)$.

Problem 3: The function $\vec{r}(t) = \langle \int_0^t \cos(\frac{1}{2}u^2)du, \int_0^t \sin(\frac{1}{2}u^2)du \rangle, t \in \mathbb{R}$ whose graph is called a **clothoid** or **Euler Spiral**, has applications in the design of railroad tracks, rollercoasters, and highways.

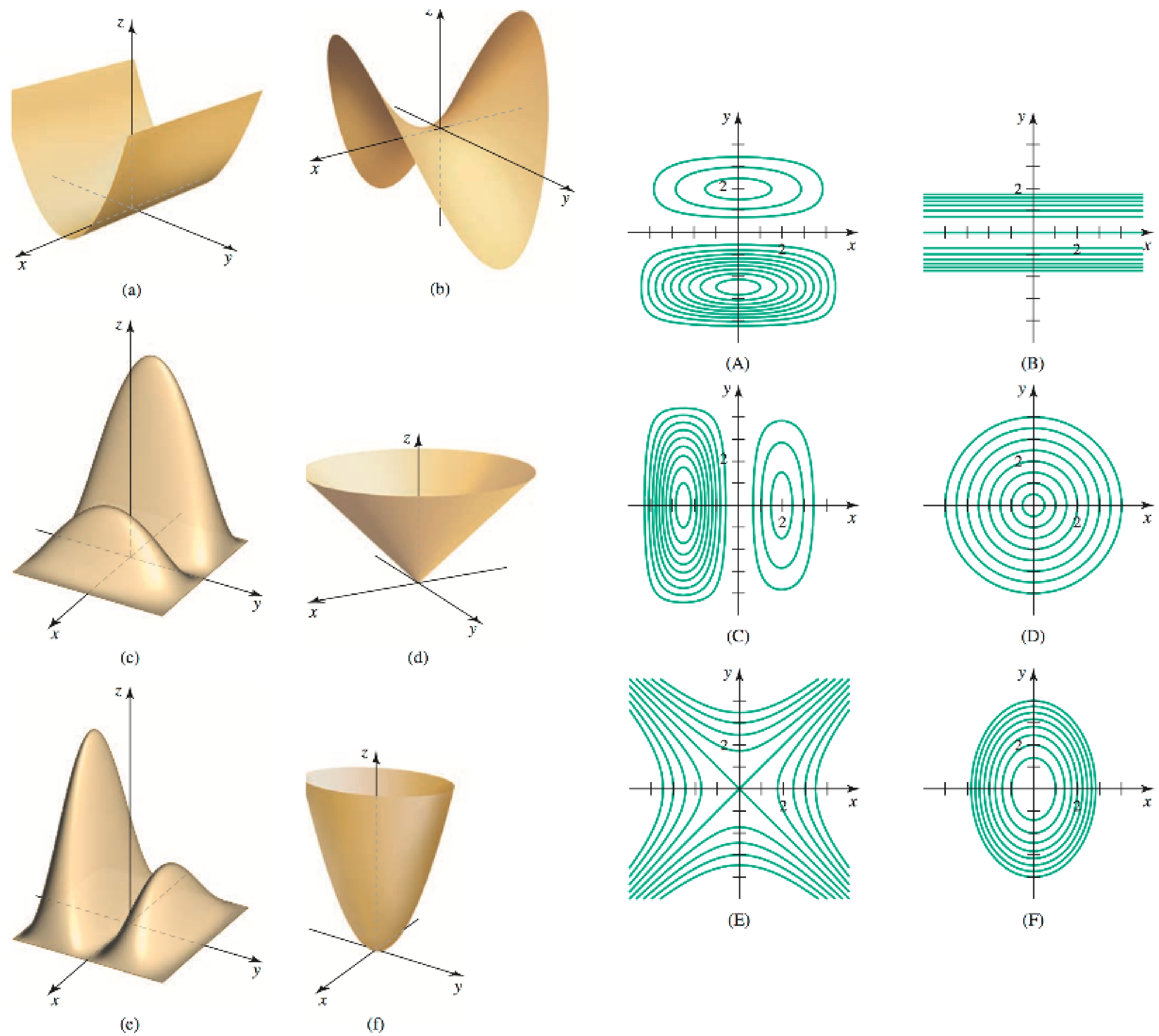


(a) A car moves from left to right on a straight highway, approaching a curve at the origin (Figure B). Sudden changes in curvature at the start of the curve may cause the driver to jerk the steering wheel. Suppose the curve starting at the origin is a segment of a circle of radius a. Explain why there is a sudden change in the curvature of the road at the origin.

(b) A better approach is to use a segment of a clothoid as an easement curve, in between the straight highway and a circle, to avoid sudden changes in curvature (Figure C). Assume the easement curve corresponds to the clothoid $\vec{r}(t)$, for $0 \le t \le 1.2$. Find the curvature of the easement curve as a function of t and explain why this curve eliminates the sudden change in curvature at the origin.

(c) Find the radius of a circle connected to the easement curve at point A (that corresponds to $t = 1.2$ on the curve $\vec{r}(t)$) so that the curvature of the circle matches the curvature of the easement curve at point A.

Problem 4: Match surfaces a-f in the figure below with level curves A-F.



Problem 5: Match functions a-d with surfaces A-D in the figure below.

a. $f(x, y) = \cos(xy)$

b. $g(x, y) = \ln(x^2 + y^2)$

c. $h(x, y) = \frac{1}{x-y}$

d. $p(x, y) = \frac{1}{1+x^2+y^2}$