Problem 1: Suppose that $\vec{u}(t)$ and $\vec{v}(t)$ are differentiable vector valued functions satisfying $\vec{u}(0) = \langle 0, 1, 1\rangle, \vec{u}\,'(0) = \langle 0, 7, 1\rangle, \vec{v}(0) = \langle 0, 1, 1\rangle$, and $\vec{v}\,'(0) = \langle 1, 1, 2\rangle$. Evaluate the following expressions.

a. $\frac{d}{dt}(\vec{u}(t) \cdot \vec{v}(t))\Big|_{t=0}$

b. $\frac{d}{dt}(\vec{u}(t) \times \vec{v}(t))\Big|_{t=0}$

c. $\frac{d}{dt}(\cos(t)\vec{u}(t))\Big|_{t=0}$

d. $\frac{d}{dt}(\vec{u}(\sin(t)))\Big|_{t=0}$

Hint: The product rules for scalar times vector, vector dot product vector, and vector cross product vector are all intuitively identical to the product rule from Calc I. Similarly, the chain rule for vector valued functions composed with scalar valued functions is intuitively identical to the chain rule from Calc I.

Problem 2: Determine whether the following statements are true or false. If a statement is true, then explain why. If a statement is false, then provide a counterexample.

(a) If the speed of an object is constant, then its velocity components are constant.

(b) The functions $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ and $\vec{R}(t) = \langle \cos(t^2), \sin(t^2) \rangle$ generate the same set of points for $t \geq 0$. (Bonus: What about for $t \geq \pi^2$?)

(c) A velocity vector (vector valued function) of variable magnitude cannot have constant direction.

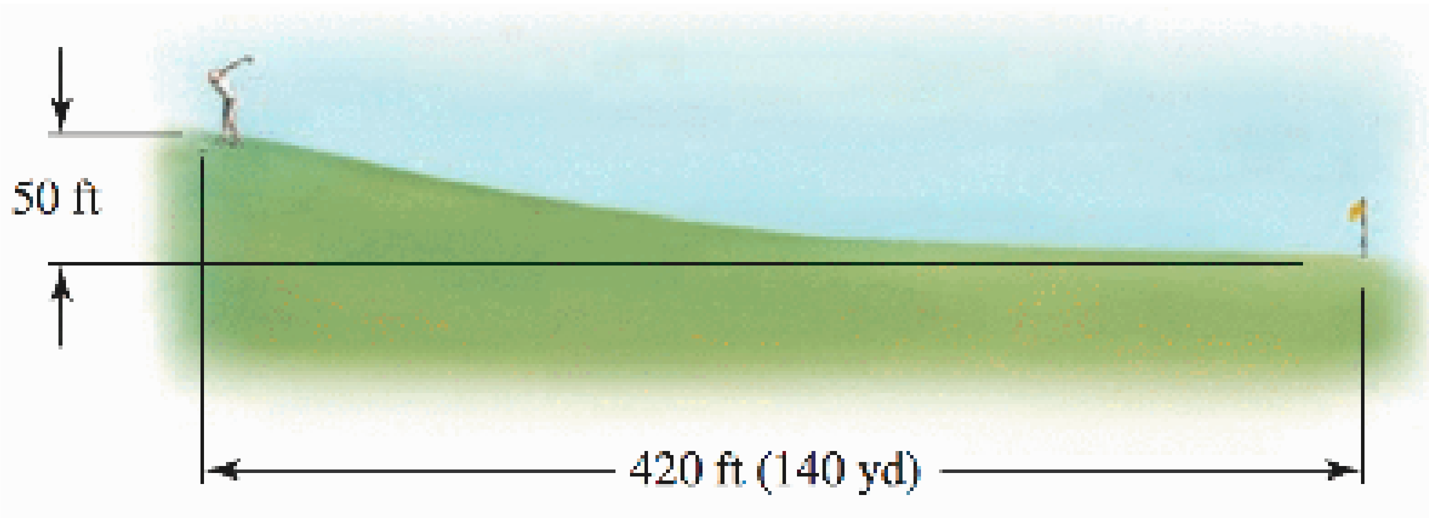
(d) If the acceleration of an object is $\vec{a}(t) = \vec{0}$, for all $t \geq 0$, then the velocity of the object is constant.

(e) If you double the initial speed of a projectile, its range also double (assume no forces other than gravity).

(f) If you double the initial speed of a projectile, its time of flight also doubles (assume no forces other than gravity).

(g) A trajectory with $\vec{v}(t) = \vec{a}(t) \neq \vec{0}$, for all t, is possible.

Problem 3: A golfer stands 420ft (140yd) horizontally from the hole and 50ft above the hole (see figure). Assuming the ball is hit with an initial speed of 120ft/s, at what angle(s) should it be hit to land in the hole? Assume the path of the ball lies in a plane. You may approximate earth's gravitational constant by $32\text{ft}/\text{s}^2$.



Problem 4: Let $\vec{r}(t) = \langle t, 2, \frac{2}{t} \rangle$ for $t > 1$. Find the unit tangent vector $\hat{T}(t)$ at all points of the curve $\vec{r}(t)$.

Note: This is intended to be an easy review problem.

Problem 5: Determine whether the following statements are true or false. If a statement is true, then explain why. If a statement is false, then provide a counterexample.

(a) If an object moves on a trajectory with constant speed S over a time interval $a \leq t \leq b$, then the length of the trajectory is $S(b-a)$.

(b) The curves defined by

$$\vec{r}(t) = \langle f(t), g(t) \rangle \text{ and } \vec{R}(t) = \langle g(t), f(t) \rangle \tag{1}$$

have the same length over the interval $[a, b]$.

(c) The curve $\vec{r}(t) = \langle f(t), g(t) \rangle$, for $0 \leq a \leq t \leq b$, and the curve $\vec{R}(t) = \langle f(t^2), g(t^2) \rangle$, for $\sqrt{a} \leq t \leq \sqrt{b}$, have the same length.

(d) The curve $\vec{r}(t) = \langle t, t^2, 3t^2 \rangle$, for $1 \leq t \leq 4$, is parameterized by arclength.

Problem 6: Consider the curve $\mathcal{C}$ that is described by the parameterization $\vec{r}(t) = \langle t^m, t^m, t^{\frac{3}{2}m}\rangle$ where $0 \le a \le t \le b$ and $m \neq 0$.

(a) Find the arclength function $s(t)$. Note that your answer may include a, b, and m.

(b) Find the parameterization by arclength for $\mathcal{C}$ when $a = \sqrt{\frac{28}{9}}, b = 4$, and $m = 2$.