Problem 1: A suitcase is pulled 50ft along a horizontal sidewalk with a constant force of 30lb at an angle of 30° above the horizontal. How much work is done?

$\|\vec{F}\| = 30\text{ lb}$

$d = 50\text{ ft}$

$$\text{Work} = \vec{F} \cdot \vec{d} = \|\vec{F}\| \cdot \|\vec{d}\| \cdot \cos(30°)$$

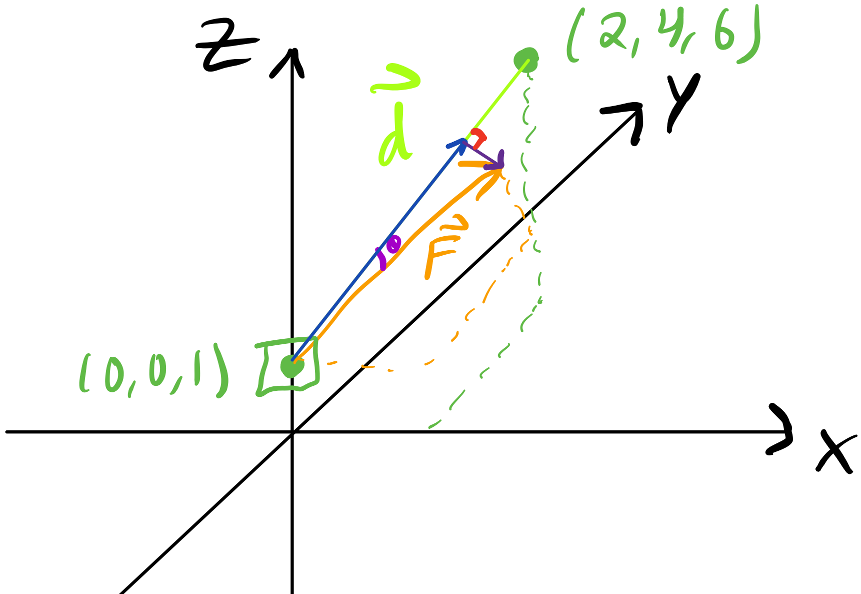
$$= 30 \cdot 50 \cdot \frac{\sqrt{3}}{2} \cdot \text{lb} \cdot \text{ft}$$

$$\text{Joules} = \underbrace{\text{Newtons}}_{\text{Force}} \cdot \underbrace{\text{meters}}_{\text{distance}}$$

$1\text{ lb} \approx 4.4482\text{ N}$

$1\text{ ft} \approx 0.3048\text{ m} \Rightarrow \approx 761.2506\text{ J}$

Problem 2: A constant force of $\vec{F} = \langle 2, 4, 1\rangle$N moves an object from $(0, 0, 1)$m to $(2, 4, 6)$m. How much work is done?

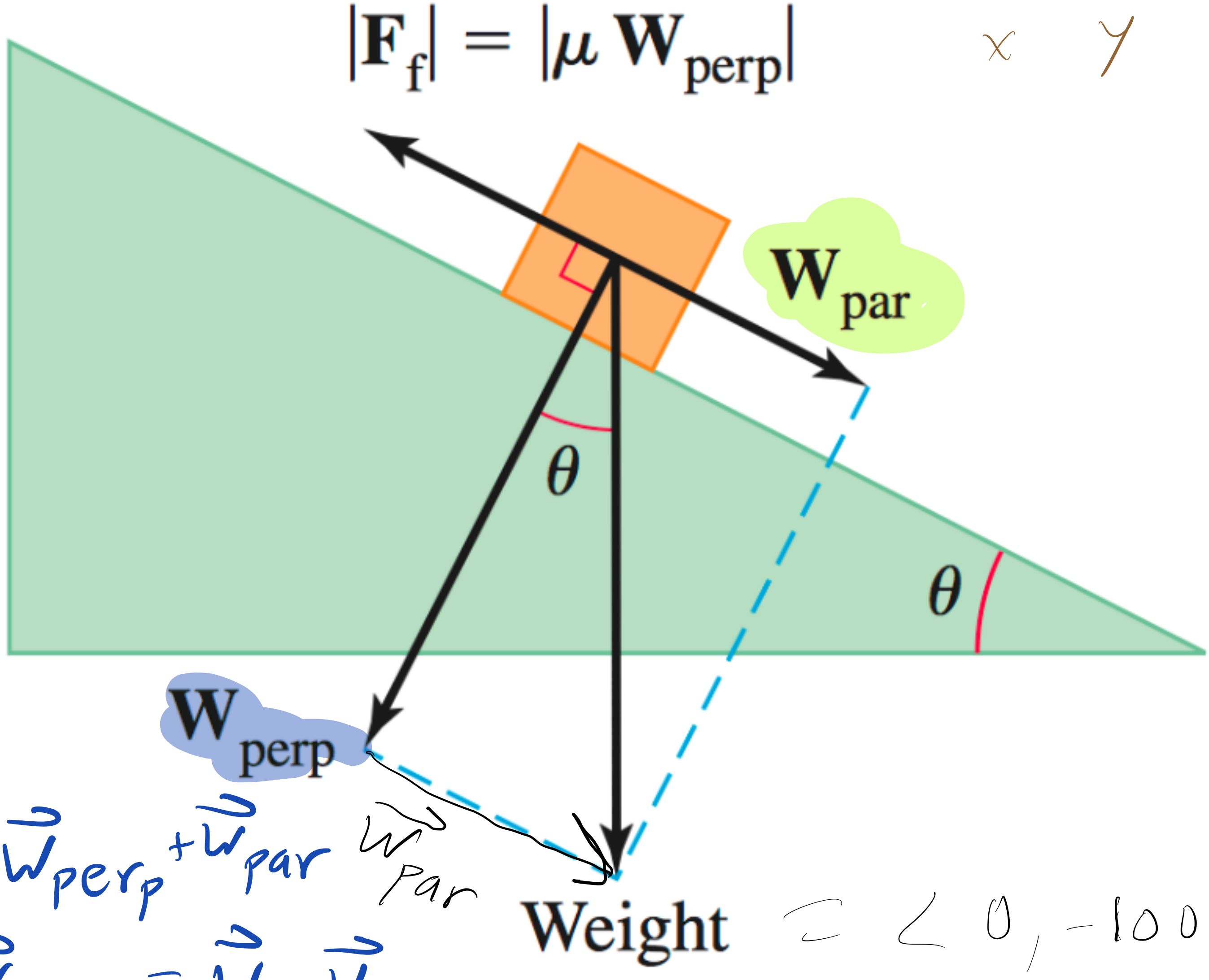


$$\vec{d} = \langle 2,4,6\rangle - \langle 0,0,1\rangle$$
$$= \langle 2,4,5\rangle$$

$$\text{Work} = \vec{F}\cdot\vec{d} = \|\vec{F}\|\cdot\|\vec{d}\|\cdot\cos(\theta)$$
$$= \langle 2,4,1\rangle\cdot\langle 2,4,5\rangle$$
$$= 2\cdot 2 + 4\cdot 4 + 1\cdot 5$$
$$= 4 + 16 + 5 = \boxed{25\,J}$$

Problem 3: An object on an inclined plane does not slide provided the component of the object's weight parallel to the plane $|\vec{W}_{\text{par}}|$ is less than or equal to the magnitude of the opposing frictional force $|\vec{F}_f|$. The magnitude of the frictional force, in turn, is proportional to the component of the object's weight perpendicular to the plane $|\vec{W}_{\text{perp}}|$. The constant of proportionality is the coefficient of static friction $\mu > 0$. Suppose a 100lb block rests on a plane that is tilted at an angle of $\theta = 30°$ to the horizontal. What is the smallest possible value of μ?

$\vec{v} = \langle 5, 7 \rangle$
x y



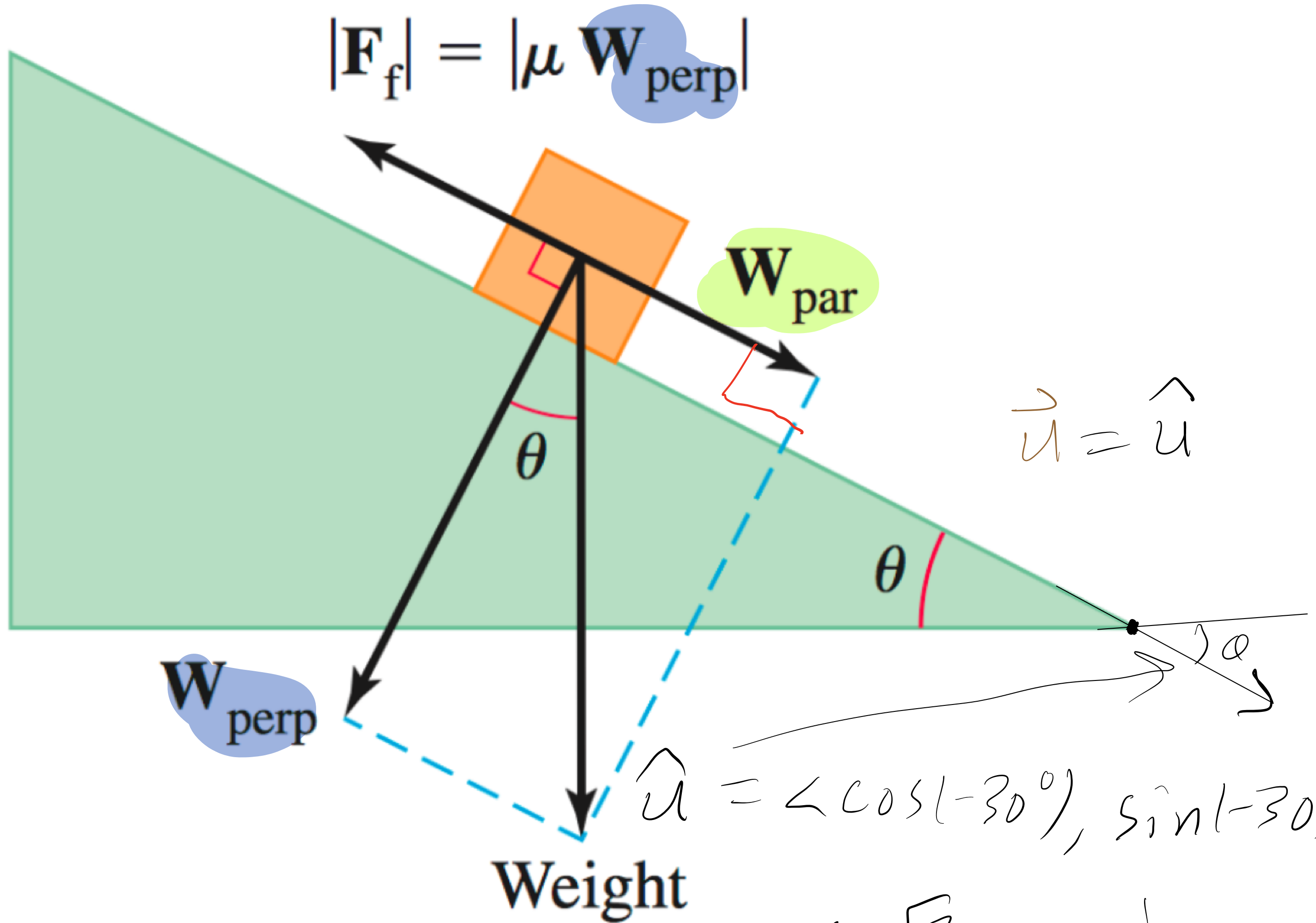
$= \langle 0, -100\text{lb} \rangle$

$\vec{W} = \vec{W}_{perp} + \vec{W}_{par}$

$\rightarrow \vec{W}_{perp} = \vec{W} - \vec{W}_{par}$

The block stays still if

$$\boxed{|\vec{W}_{par}| = |\vec{F}_f| = \mu |\vec{W}_{perp}|}$$



$\vec{u} = \hat{u}$

$\hat{u} = \langle \cos(-30°), \sin(-30°) \rangle$

$= \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$

$$\vec{W}_{par} = \mathrm{Proj}_{\hat{u}} \vec{w} = \left(\frac{\vec{w} \cdot \hat{u}}{\hat{u} \cdot \hat{u}} \right) \hat{u}$$

$$= (\vec{w} \cdot \hat{u}) \hat{u}$$

$\hat{u} \cdot \hat{u} = \|\hat{u}\|^2 = 1^2 = 1$

$$= \left(\langle 0, -100 \rangle \cdot \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle \right) \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$$

$$= \left(0 \cdot \frac{\sqrt{3}}{2} + (-100) \cdot (-\frac{1}{2}) \right) \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$$

$$= 50\left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle = \langle 25\sqrt{3}, -25 \rangle$$

$$\vec{w}_{perp} = \vec{w} - \vec{w}_{par}$$

$$= \langle 0, -100 \rangle - \langle 25\sqrt{3}, -25 \rangle$$

$$= \langle 0 - 25\sqrt{3}, -100 - (-25) \rangle$$

$$= \langle -25\sqrt{3}, -75 \rangle$$

$$\boxed{\mu = \frac{|\vec{w}_{par}|}{|\vec{w}_{perp}|}} = \frac{50}{50\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$|\vec{w}_{par}| = \sqrt{(25\sqrt{3})^2 + (-25)^2}$$

$$= 25\sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$= 25\sqrt{3+1} = 25 \cdot 2 = 50$$

$$|\vec{w}_{perp}| = \sqrt{(-25\sqrt{3})^2 + (-75)^2}$$

$$= 25\sqrt{(-\sqrt{3})^2 + (-3)^2}$$

$$= 25\sqrt{3+9} = 25\sqrt{12}$$

$$= 50\sqrt{3}$$

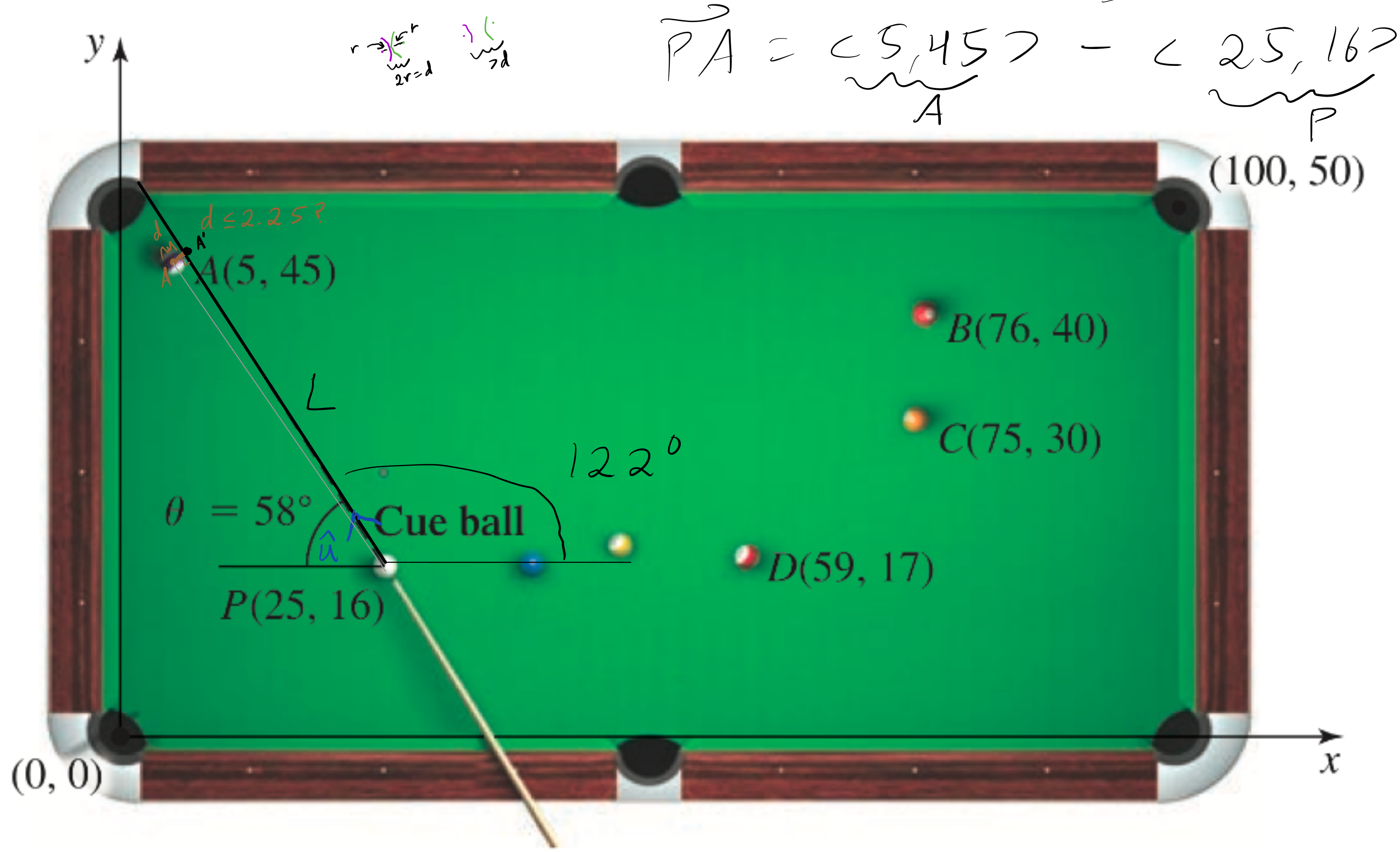
Problem 4: A cue ball in a billiards video game lies at P(25, 16). We assume that each ball has a diameter of 2.25 screen units, and pool balls are represented by the point at their center.

a) The cue ball is aimed at an angle of 58° above the negative x-axis toward a target ball at A(5, 45). Do the balls collide?

b) The cue ball is aimed at the point (50, 25) in an attempt to hit a target ball at B(76, 40). Do the balls collide?

c) The cue ball is aimed at an angle θ above the x-axis in the general direction of a target ball at C(75, 30). What range of angles (for $0 \le \theta \le \frac{\pi}{2}$) will result in a collision? Express your answer in degrees.

$$\vec{PA} = \underbrace{\langle 5, 45 \rangle}_{A} - \underbrace{\langle 25, 16 \rangle}_{P} = \langle -20, 29 \rangle$$



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a) $\hat{u} = \langle \cos(122^\circ), \sin(122^\circ) \rangle$

$L = L(t) = \vec{P} + t\hat{u} = \langle 25, 16 \rangle + t\hat{u}$

$= \langle 25 + t\cos(122^\circ), 16 + t\sin(122^\circ) \rangle$

$$\vec{PA'} = \operatorname{Proj}_{\hat{u}} \vec{PA} = \left(\frac{\vec{PA}\cdot\hat{u}}{\hat{u}\cdot\hat{u}}\right)\hat{u} = (\vec{PA}\cdot\hat{u})\hat{u}$$

"L"

$$= (\langle -20, 29\rangle \cdot \langle \cos(122^\circ), \sin(122^\circ)\rangle)\langle \cos(122^\circ), \sin(122^\circ)\rangle$$

$$\approx \langle -18.65, 29.84\rangle$$

$$\vec{AA'} = \vec{PA} - \vec{PA'} \approx \langle -20, 29\rangle - \langle -18.65, 29.84\rangle$$

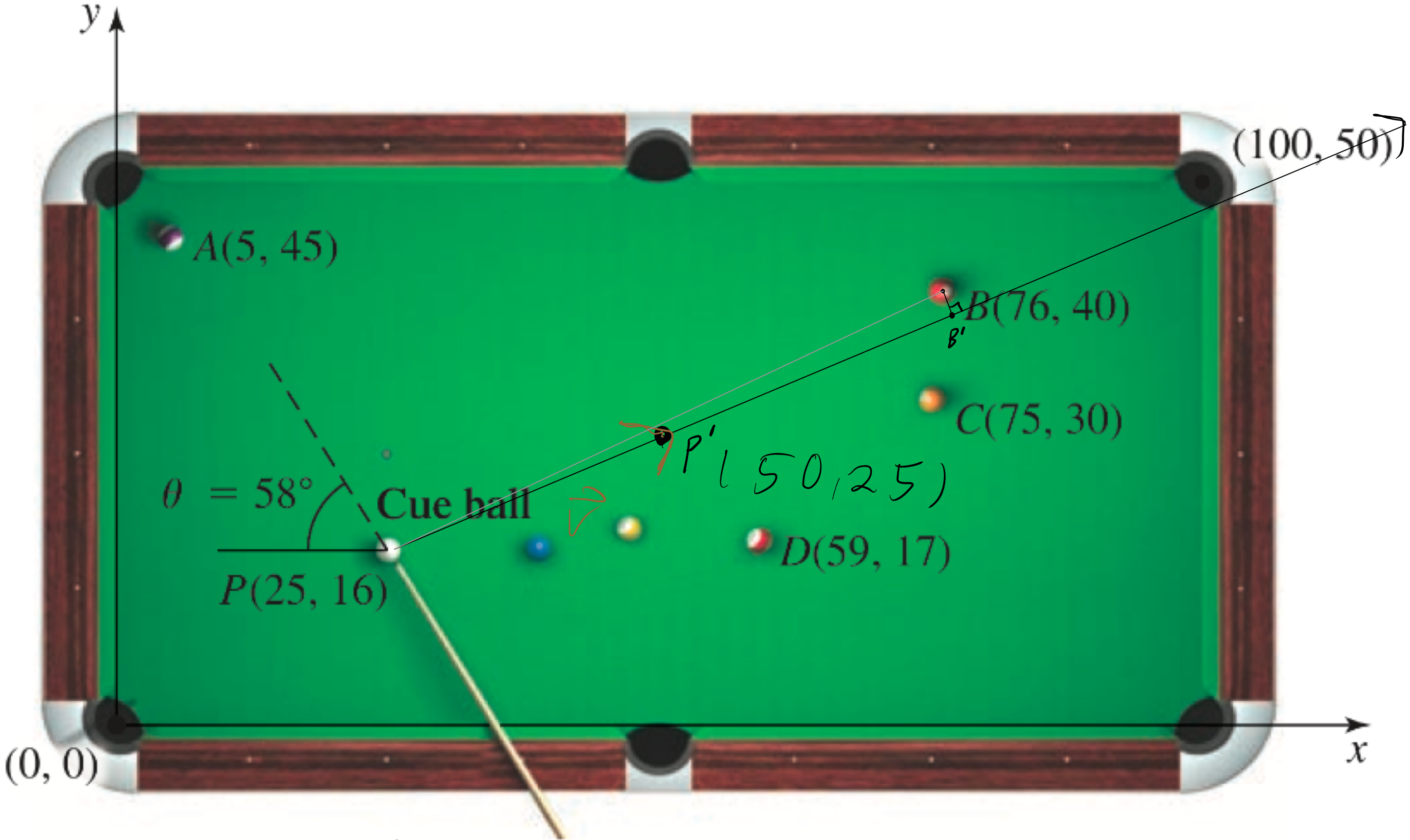
$$= \langle -1.35, -0.84\rangle$$

$$\rightarrow |\vec{AA'}| = \sqrt{(-1.35)^2 + (-0.84)^2}$$

$$= 1.59 < 2.25$$

$\rightarrow$ The balls collide

b)



$$\vec{v} = \overrightarrow{PP'} = \underbrace{\langle 50,25\rangle}_{P'} - \underbrace{\langle 25,16\rangle}_{P} = \langle 25,9\rangle$$

$$\overrightarrow{PB} = \underbrace{\langle 76,40\rangle}_{B} - \underbrace{\langle 25,16\rangle}_{P}$$

$$= \langle 51,24\rangle$$

$$\overrightarrow{PB'} = \mathrm{Proj}_{\vec{v}}\overrightarrow{PB} = \left(\frac{\overrightarrow{PB}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$$

$$= \left(\frac{\langle 51, 24 \rangle \cdot \langle 25, 9 \rangle}{\langle 25, 9 \rangle \cdot \langle 25, 9 \rangle} \right) \langle 25, 9 \rangle$$

$$= \left(\frac{51 \cdot 25 + 24 \cdot 9}{25 \cdot 25 + 9 \cdot 9} \right) \langle 25, 9 \rangle$$

$$\approx \langle 52.80, 19.01 \rangle$$

$$\overrightarrow{BB'} = \overrightarrow{PB'} - \overrightarrow{PB}$$

$$\approx \langle 51, 24 \rangle - \langle 52.80, 19.01 \rangle$$

$$= \langle -1.80, 4.99 \rangle$$

$$\rightarrow |\overrightarrow{BB'}| \geq 4.99 > 2.25$$

$\rightarrow$ The balls do not collide

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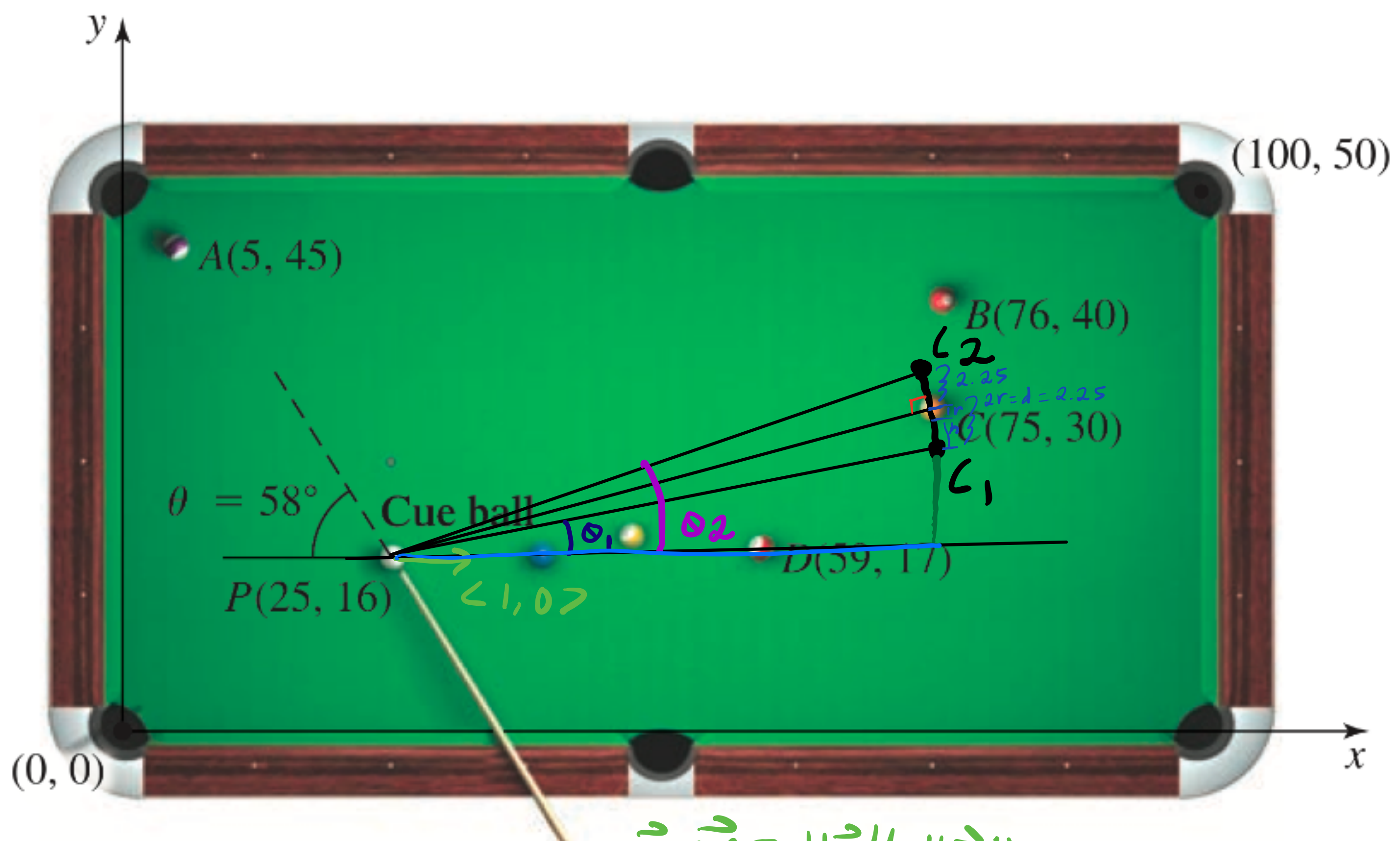

$\vec{u} \cdot \vec{w} = ||\vec{u}|| \cdot ||\vec{w}|| \cdot \cos(\theta)$

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c) $\overrightarrow{PC} = \underbrace{\langle 75, 30 \rangle}_{C} - \underbrace{\langle 25, 16 \rangle}_{P} = \langle 50, 14 \rangle$

$\vec{v} = \langle -14, 50 \rangle \perp \overrightarrow{PC}$

Fact: $\langle x, y\rangle \perp \langle -y, x\rangle$ b/c

$$\langle x, y\rangle \cdot \langle -y, x\rangle = x\cdot(-y) + yx = 0 \checkmark$$

and $\langle x, y\rangle \perp \langle y, -x\rangle$ ......

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle -14, 50\rangle}{\sqrt{(-14)^2 + 50^2}}$$

$$PC_2 = PC + 2.25\,\hat{v} \approx \langle 49.39, 16.17\rangle$$

$$PC_1 = PC - 2.25\,\hat{v} \approx \langle 50.61, 11.83\rangle$$

$$\theta_2 = \tan^{-1}\left(\frac{16.17}{49.39}\right) \approx 18.13^\circ$$

$$\theta_1 = \tan^{-1}\left(\frac{11.83}{50.61}\right) \approx 13.16^\circ$$

$\rightarrow$ $13.16^\circ \leq \theta \leq 18.13^\circ$