Quadric Surfaces

a quadric surface is defined by

$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$$

E.g. 1. $x^2+y^2+z^2-4x+2y+6z+13=0$

$\to \quad \overbrace{(x-2)^2}^{x^2} \overset{+}{+} \overbrace{(y+1)^2}^{y^2} \overset{+}{+} \overbrace{(z+3)^2}^{z^2} \overset{=r^2}{=} 1 = 1^2$

$\to$ Sphere of radius 1 centered at $(2,-1,-3)$

E.g. 2. $z^2+x^2-2xy+y^2-1=0$

$\to \quad z^2+(x-y)^2=1 \to$ tilted cylinder of radius 1.

Ellipsoid (centered at (0,0,0))

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$a=b=c \to$ Sphere of radius $r=a=b=c$

$(1, 2, 3)$

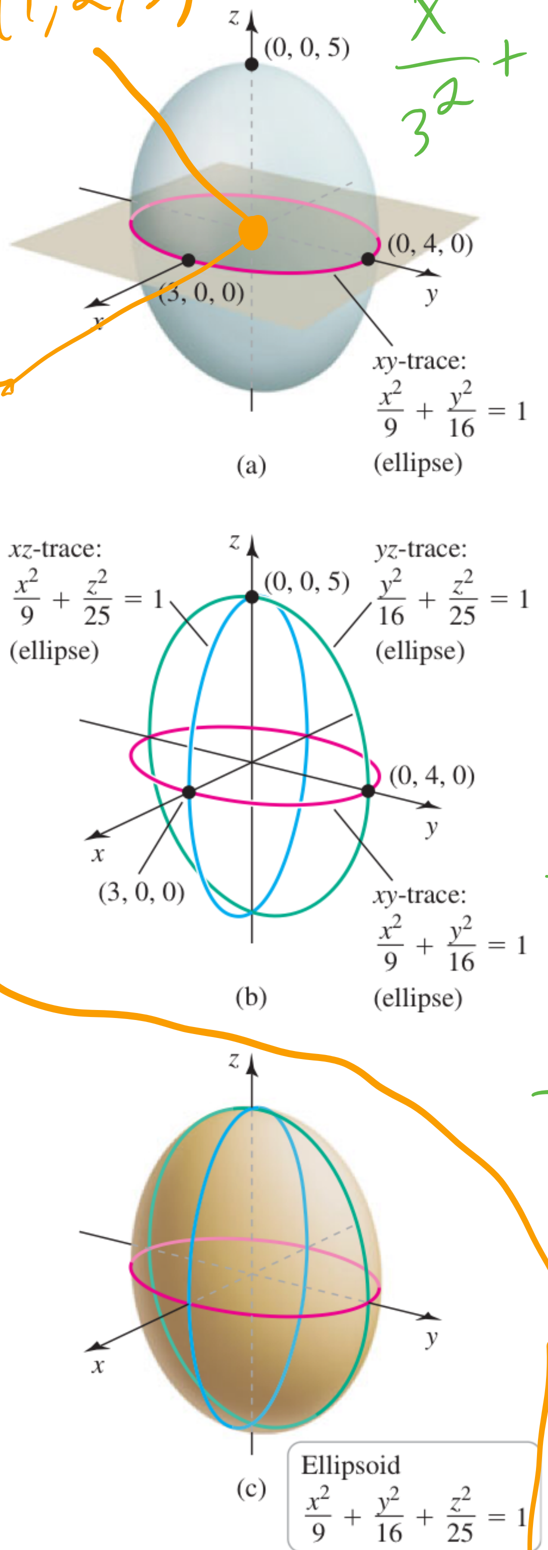


FIGURE 13.13

$$\frac{x^2}{3^2} + \frac{y^2}{4^2} + \frac{z^2}{5^2} = 1$$

Suppose $z = z_0 = 1$

$$\frac{x^2}{3^2} + \frac{y^2}{4^2} + \frac{1}{5^2} = 1$$

$$\rightarrow \frac{x^2}{3^2} + \frac{y^2}{4^2} = \frac{24}{25}$$

$$\rightarrow \frac{x^2}{3^2 \cdot \frac{24}{25}} + \frac{y^2}{4^2 \cdot \frac{24}{25}} = 1$$

$$\rightarrow \frac{x^2}{\left(3 \frac{\sqrt{24}}{5}\right)^2} + \frac{y^2}{\left(4 \frac{\sqrt{24}}{5}\right)^2} = 1$$

$$\boxed{\frac{(x-1)^2}{3^2} + \frac{(y-2)^2}{4^2} + \frac{(z-3)^2}{5^2}}$$

Elliptic Paraboloid ("centered" at (0,0,0))

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$1 = \frac{x^2}{16z_0} + \frac{y^2}{4z_0}$$

$$= \frac{x^2}{(4\sqrt{z_0})^2} + \frac{y^2}{(2\sqrt{z_0})^2}$$

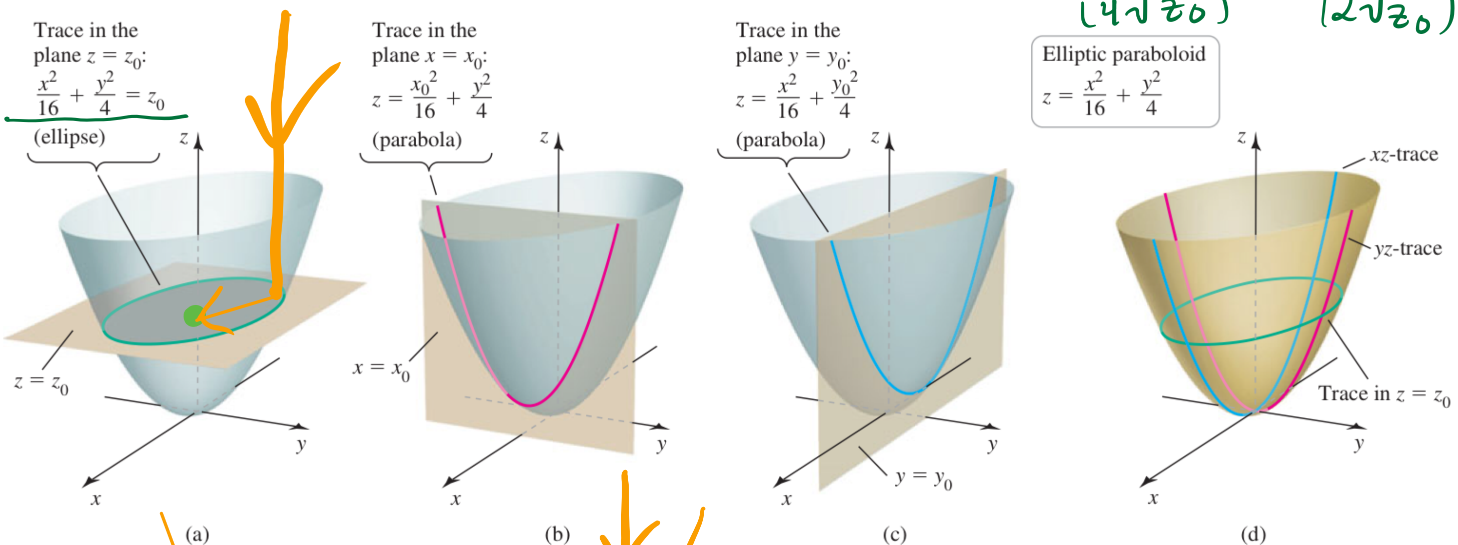
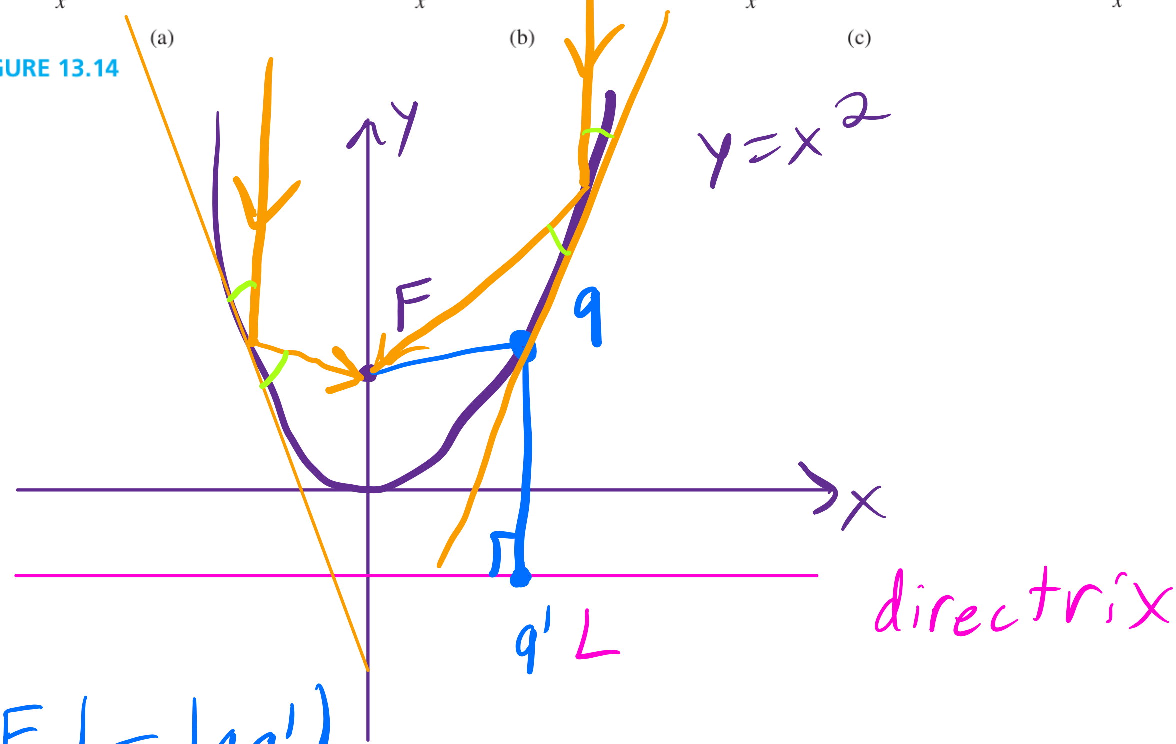
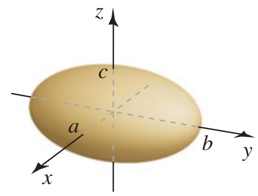
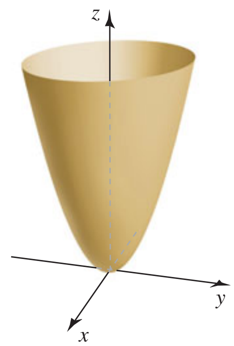
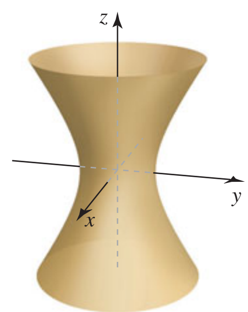
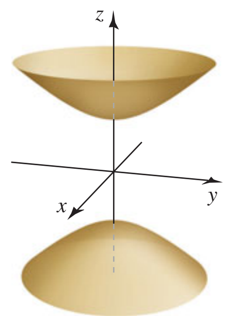
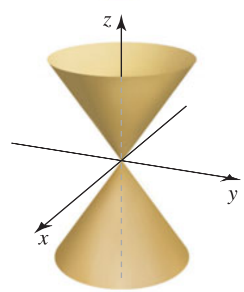


FIGURE 13.14



$$|Fq| = |qq'|$$

Table 13.1

Name	**Standard Equation**	**Features**	**Graph**
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ with $\lvert z_0 \rvert > \lvert c \rvert$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$	Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.