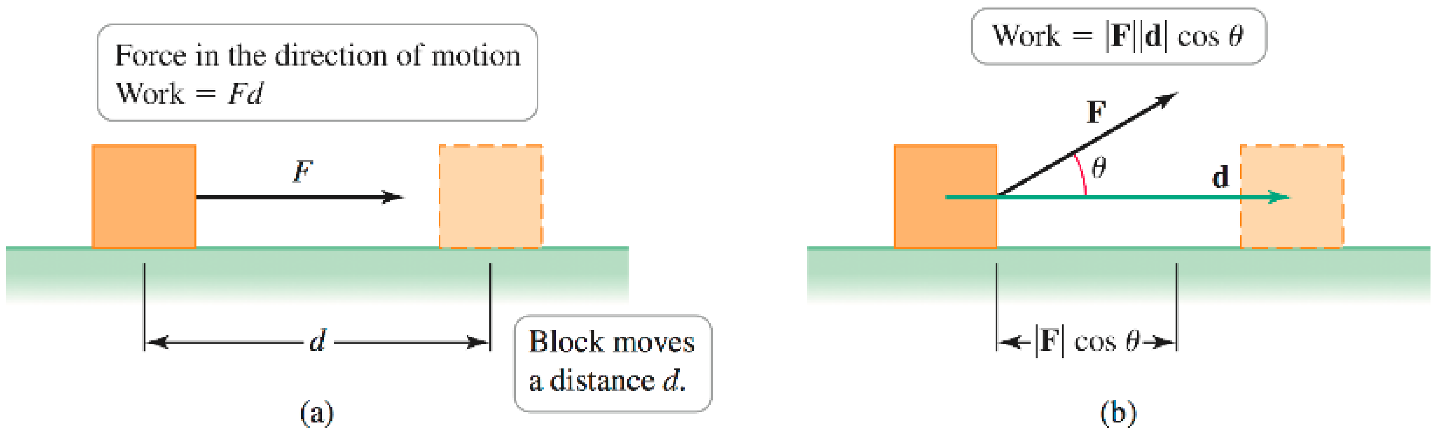
Problem 13.3.4.4: A suitcase is pulled 50ft along a horizontal sidewalk with a constant force of 30lb at an angle of 30° above the horizontal. How much work is done?

Solution: For this problem it suffices to use the formula for work that is shown in the diagram below.



The only thing that we need to be careful of is to remember that the standard unit of measure for work is Joules (J) which is given by $\text{J} = \text{kg}\cdot\text{m}/s^2 = \text{N}\cdot\text{m}$, where N represents Newtons. To this end, we recall that 1lb≈4.4482N and 1ft≈0.3048m. It follows that the total amount of work done is given by

$$\text{Work} = 30\text{lb} \cdot 50\text{ft} \cdot \cos(30^\circ) \approx 30 \cdot 4.4482\text{N} \cdot 50 \cdot 0.3048\text{m} \cdot \frac{\sqrt{3}}{2} \approx \boxed{761.2506\text{J}}. \tag{1}$$

Modified Problem 13.3.4.46: A constant force of $\vec{F} = \langle 2, 4, 1\rangle$N moves an object from $(0, 0, 1)$m to $(2, 4, 6)$m. How much work is done?

Solution: For this problem it helps to use the formula

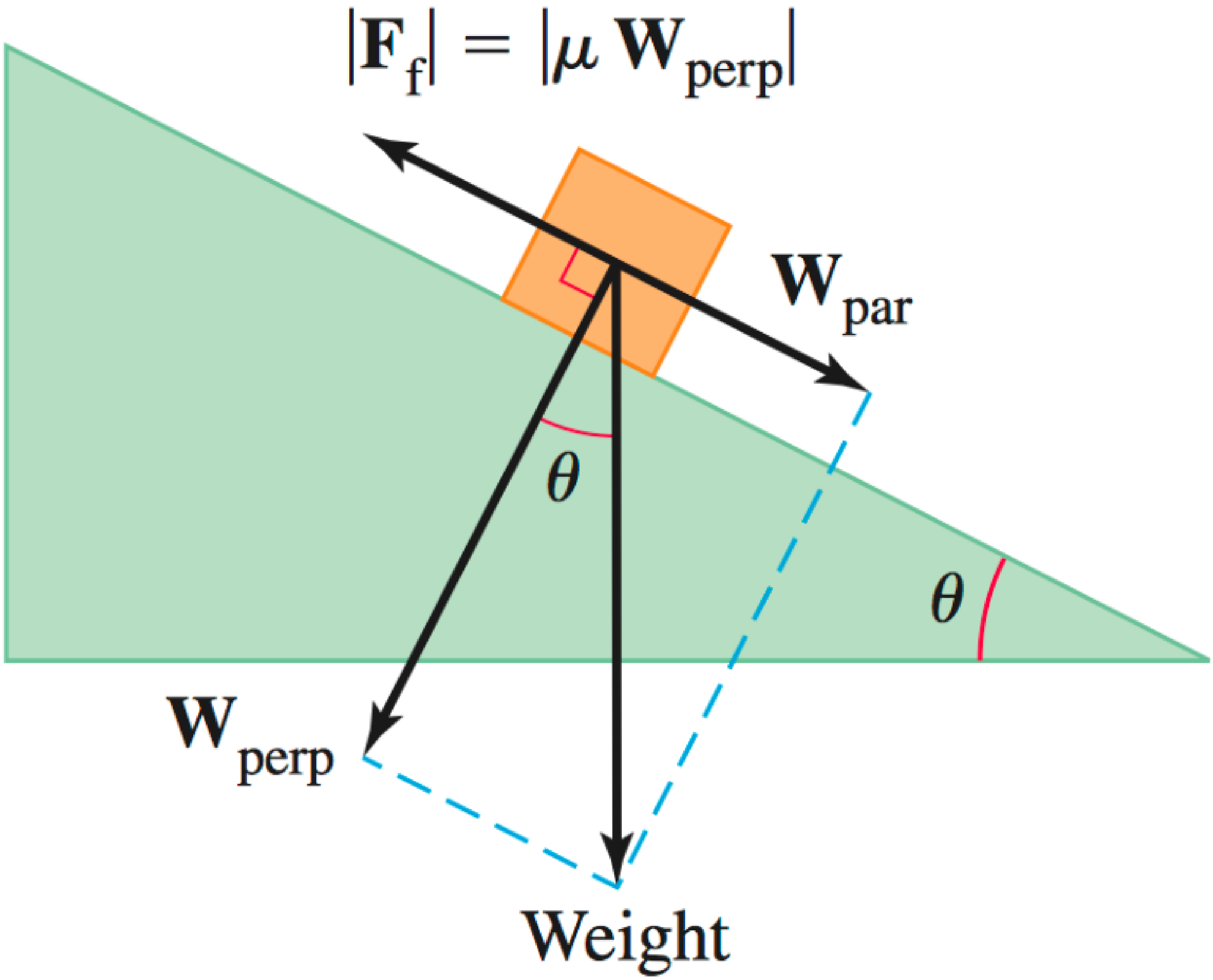
$$\text{Work} = |\vec{F}| \cdot |\vec{d}\,| \cos(\theta) = \vec{F} \cdot \vec{d}, \tag{2}$$

where $\vec{F}$ is a constant force that is applied to an object that moves in a straight line with a final displacement of $\vec{d}$. We now see that

$$\vec{d} = \langle 2, 4, 6\rangle m - \langle 0, 0, 1\rangle m = \langle 2, 4, 5\rangle m \tag{3}$$

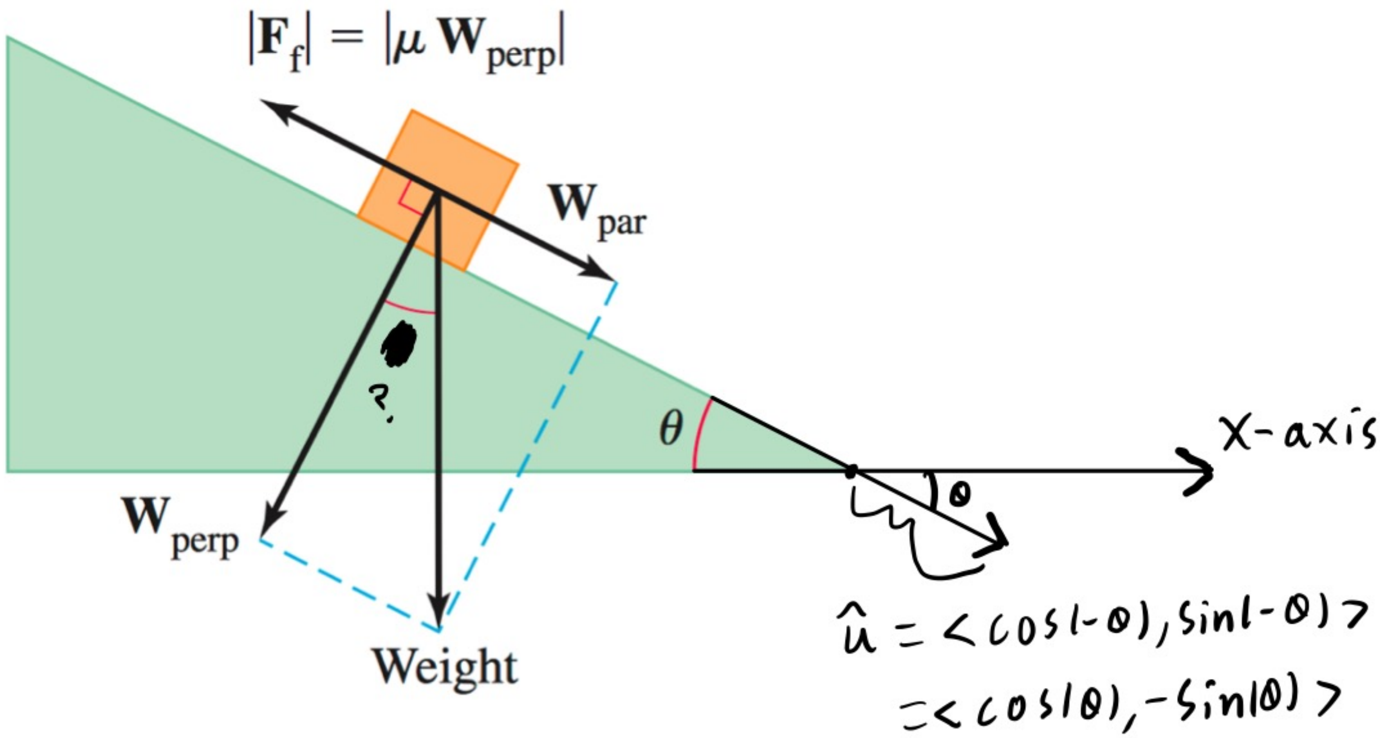
$$\rightarrow \text{Work} = \underbrace{\langle 2, 4, 1\rangle N}_{\vec{F}} \cdot \underbrace{\langle 2, 4, 5\rangle m}_{\vec{d}} = \boxed{25\text{J}}. \tag{4}$$

Modified Problem 13.3.4.50: An object on an inclined plane does not slide provided the component of the object's weight parallel to the plane $|\mathbf{W}_{\text{par}}|$ is less than or equal to the magnitude of the opposing frictional force $|\mathbf{F}_f|$. The magnitude of the frictional force, in turn, is proportional to the component of the object's weight perpendicular to the plane $|\mathbf{W}_{\text{perp}}|$. The constant of proportionality is the coefficient of static friction $\mu > 0$. Suppose a 100lb block rests on a plane that is tilted at an angle of $\theta = 30°$ to the horizontal. What is the smallest possible value of μ?



We will present 2 solutions to this problem. The first solution is a direct approach but is computationally intensive. The second solution requires a little more ingenuity but is shorter. For the sake of generality, in both solutions we will solve the problem for a general angle θ and weight w and only plug in $\theta = 30°$ and $w = 100$ at the very end.

Solution 1: We see that $\mathbf{W} = \mathbf{W}_{\text{par}} + \mathbf{W}_{\text{perp}}$ is an decomposition of the force of gravity $\mathbf{W}$ into a sum of two orthogonal components. Since we know that $\mathbf{W} = \langle 0, -w\rangle$lb we only need to find $\mathbf{W}_{\text{par}}$ and it will then be easy to obtain $\mathbf{W}_{\text{perp}}$ through subtraction. To find $\mathbf{W}_{\text{par}}$ we calculate the orthogonal projection of $\mathbf{W}$ onto $\hat{u}$, the direction of the ramp as shown in the diagram below.



We now see that

$$\mathbf{W}_{\text{par}} = \text{Proj}_{\hat{u}}\mathbf{W} = \frac{\mathbf{W}\cdot\hat{u}}{|\hat{u}|^2}\hat{u} = (\mathbf{W}\cdot\hat{u})\hat{u} \tag{5}$$

$$= (\langle 0, -w\rangle \cdot \langle\cos(\theta), -\sin(\theta)\rangle)\langle\cos(\theta), -\sin(\theta)\rangle \tag{6}$$

$$= \langle w\sin(\theta)\cos(\theta), -w\sin(\theta)^2\rangle \tag{7}$$

$$\mathbf{W}_{\text{perp}} = \mathbf{W} - \mathbf{W}_{\text{par}} = \langle 0, -w\rangle - \langle w\sin(\theta)\cos(\theta), -w\sin(\theta)^2\rangle \tag{8}$$

$$= \langle -w\sin(\theta)\cos(\theta), w(-1+\sin^2\theta)\rangle = \langle -w\sin(\theta)\cos(\theta), -w\cos^2(\theta)\rangle \tag{9}$$

We now recall that we are searching for μ for which

$$(10)\qquad |\mathbf{W}_{\text{par}}| = |\mathbf{F}_{\text{f}}| = |\mu\mathbf{W}_{\text{perp}}| = \mu|\mathbf{W}_{\text{perp}}| \to \mu = \frac{|\mathbf{W}_{\text{par}}|}{|\mathbf{W}_{\text{perp}}|}$$

To this end, we see that[1]

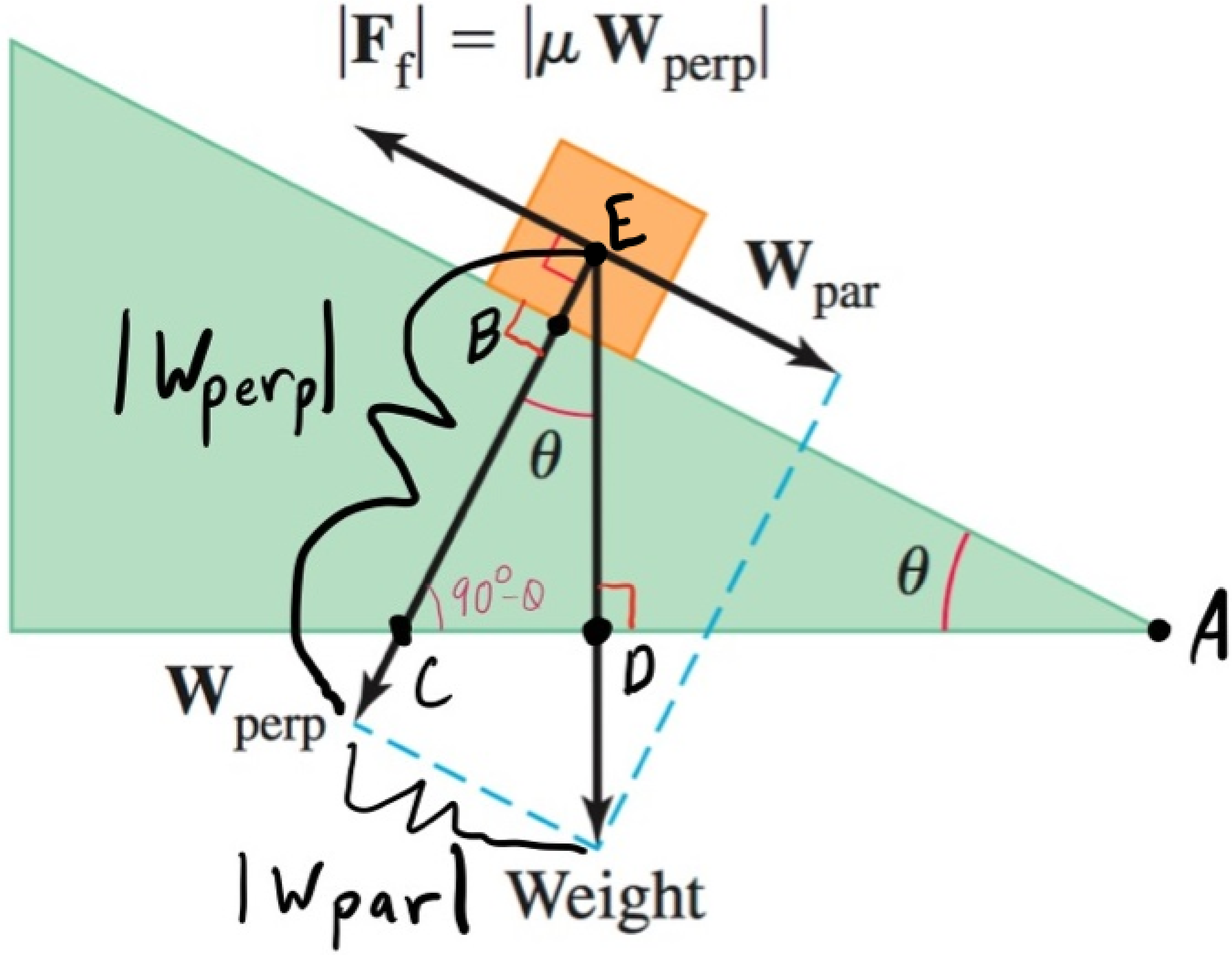
$$(11)\quad |\mathbf{W}_{\text{par}}| = \sqrt{(w\sin(\theta)\cos(\theta))^2 + (-w\sin(\theta)^2)^2}$$
$$= w\sin(\theta)\sqrt{\cos^2(\theta)+\sin^2(\theta)} = w\sin(\theta), \text{ and}$$

$$(12)\quad |\mathbf{W}_{\text{perp}}| = \sqrt{(-w\sin(\theta)\cos(\theta))^2 + (-w\cos^2(\theta))^2}$$
$$= w\cos(\theta)\sqrt{\sin^2(\theta)+\cos^2(\theta)} = w\cos(\theta), \text{ hence}$$

$$(13)\qquad \mu = \frac{|\mathbf{W}_{\text{par}}|}{|\mathbf{W}_{\text{perp}}|} = \frac{w\sin(\theta)}{w\cos(\theta)} = \boxed{\tan(\theta)} = \tan(30^\circ) = \boxed{\frac{1}{\sqrt{3}}}.$$

Solution 2: First, let us verify that the two angles labeled with θ in the given diagram are indeed the same angle. We begin by labeling points on the original diagram as shown in the new diagram below in order to obtain the subsequent calculations.

[1] Recall that $\sin(\theta), \cos(\theta) \geq 0$ when $0 \leq \theta \leq 90^\circ$, so we don't need to write $|\sin(\theta)|$ or $|\cos(\theta)|$ in this case.



(14) $$\angle ACB = 90° - \angle BAC = 90° - \theta \text{ and}$$

(15) $$\angle CED = 90° - \angle DCE = 90° - \angle ACB = 90° - (90° - \theta) = \theta,$$

so the given diagram was indeed correctly labeled. We now recall that we are searching for μ for which

(16) $$|\mathbf{W}_{\text{par}}| = |\mathbf{F}_{\text{f}}| = |\mu\mathbf{W}_{\text{perp}}| = \mu|\mathbf{W}_{\text{perp}}| \to \mu = \frac{|\mathbf{W}_{\text{par}}|}{|\mathbf{W}_{\text{perp}}|}$$

After taking a look at our labeled diagram we see that

(17) $$\frac{|\mathbf{W}_{\text{par}}|}{|\mathbf{W}_{\text{perp}}|} = \boxed{\tan(\theta)} = \tan(30°) = \boxed{\frac{1}{\sqrt{3}}}.$$