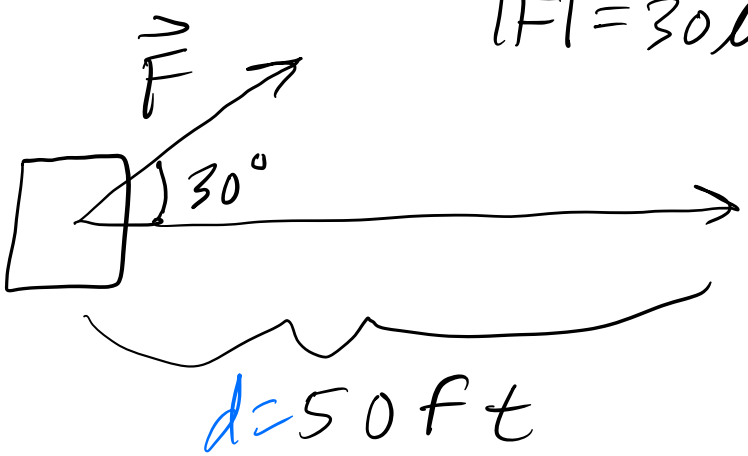
13.3.4.4) A suitcase is pulled 50ft along a horizontal sidewalk with constant force 30 lb at an angle 30° above the horizontal. How much work is done?

Solution:



$$\text{Work} = |\vec{F}|\cos(\theta)d$$

Work = Force times distance (displacement)

$= 30\,lb \cdot \cos(30^\circ) \cdot 50\,ft$

$= 1500 \cdot \frac{\sqrt{3}}{2}\,lb \cdot ft$

$= 750\sqrt{3}\,lb \cdot ft$

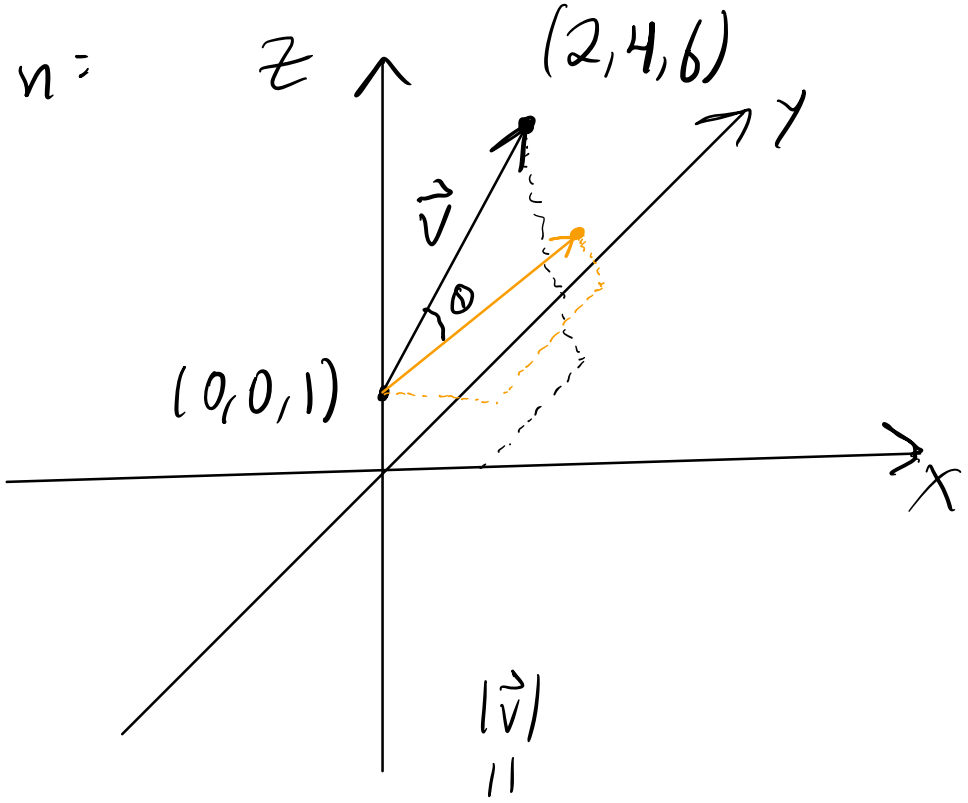
Work (Joules) =

Force (Newtons) · distance (meters)

$\approx 750\sqrt{3}\,(4.448\,N)\,(0.3048\,M)$

13.3.4. "46") A constant force $\vec{F} = \langle 2, 4, 1 \rangle N$ moves an object from $(0, 0, 1)m$ to $(2, 4, 6)m$. How much work is done?

Solution:



$$\text{Work} = |\vec{F}|\cos|\theta|d = \vec{F} \cdot \vec{v}$$

$\vec{v} = \langle 2, 4, 6 \rangle - \langle 0, 0, 1 \rangle$

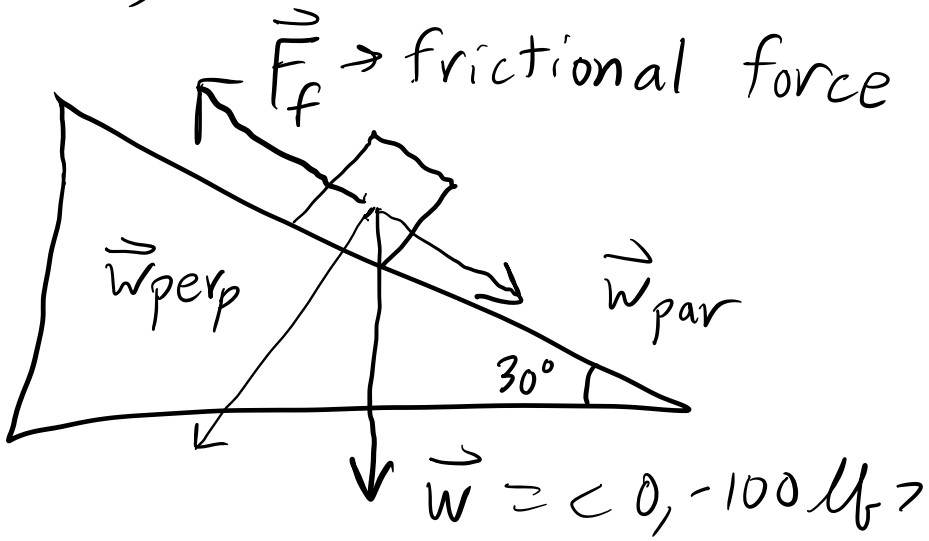
$= \langle 2, 4, 5 \rangle$

$\vec{F} \cdot \vec{v} = \langle 2, 4, 1 \rangle \cdot \langle 2, 4, 5 \rangle$

$= 2 \cdot 2 + 4 \cdot 4 + 1 \cdot 5 = 4 + 16 + 5$

$= \boxed{25 \text{ J}}$

13.3.4. "50")

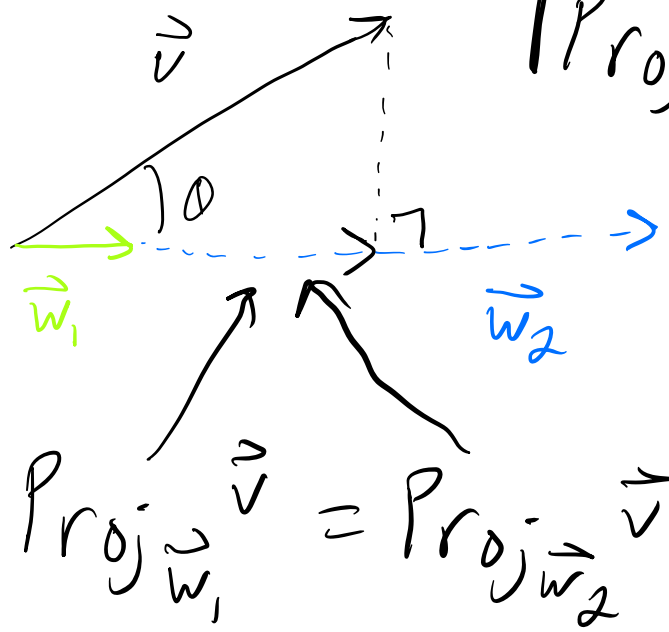


μ is the coeficient of friction.

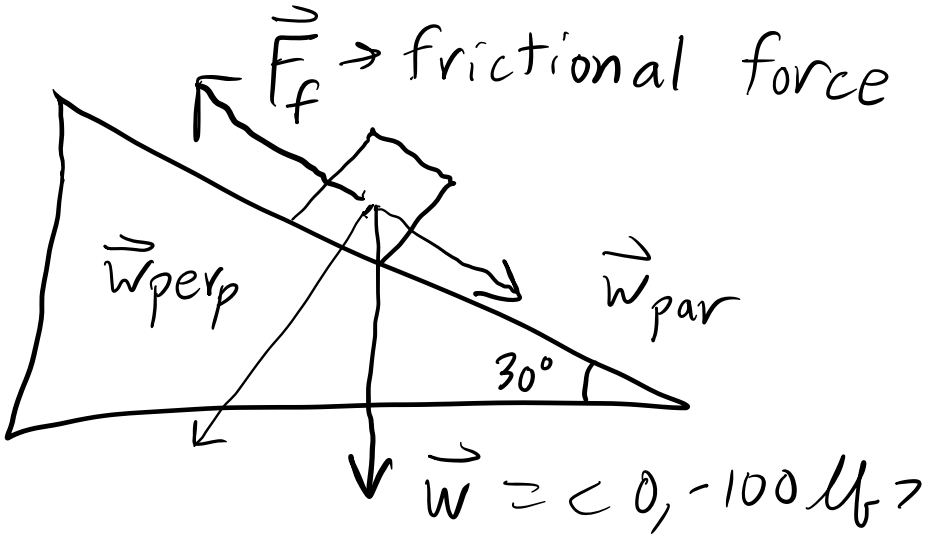
$$|\vec{F}_f| = \mu |\vec{w}_{perp}|.$$

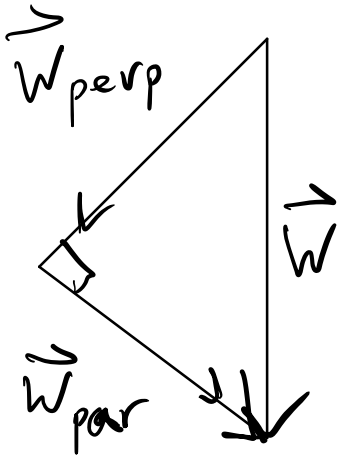
Find μ s.t. (such that) the block does not slide down the ramp.

Solution:

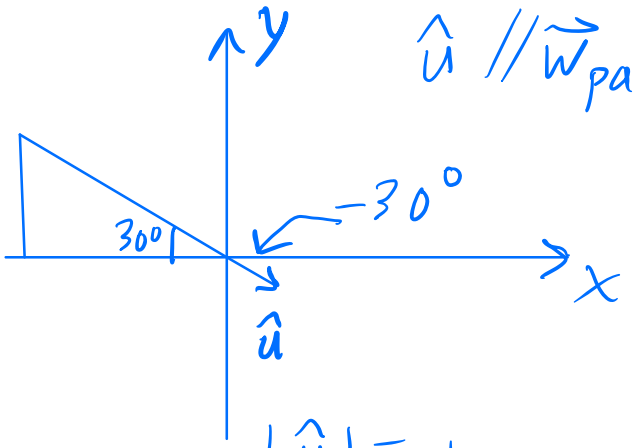


$$|\text{Proj}_{\vec{w}_1}\vec{v}| = |\vec{v}|\cos(\theta)$$





$\vec{w} = \vec{w}_{perp} + \vec{w}_{par}$



$\hat{u} \parallel \vec{w}_{par}$

$|\hat{u}| = 1$

$\hat{u} = \langle \cos(-30°), \sin(-30°) \rangle$

$= \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$

$$\vec{w}_{par} = \text{Proj}_{\vec{w}_{par}} \vec{w} = \text{Proj}_{\hat{u}} \vec{w}$$

$$= \frac{\hat{u}\cdot\vec{w}}{|\hat{u}|^2}\hat{u} = (\hat{u}\cdot\vec{w})\hat{u}$$

$$= \left(\left\langle \tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}\right\rangle \cdot \langle 0, -100\rangle\right)\left\langle \tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}\right\rangle$$

$$= (0 + 50)\left\langle \tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}\right\rangle$$

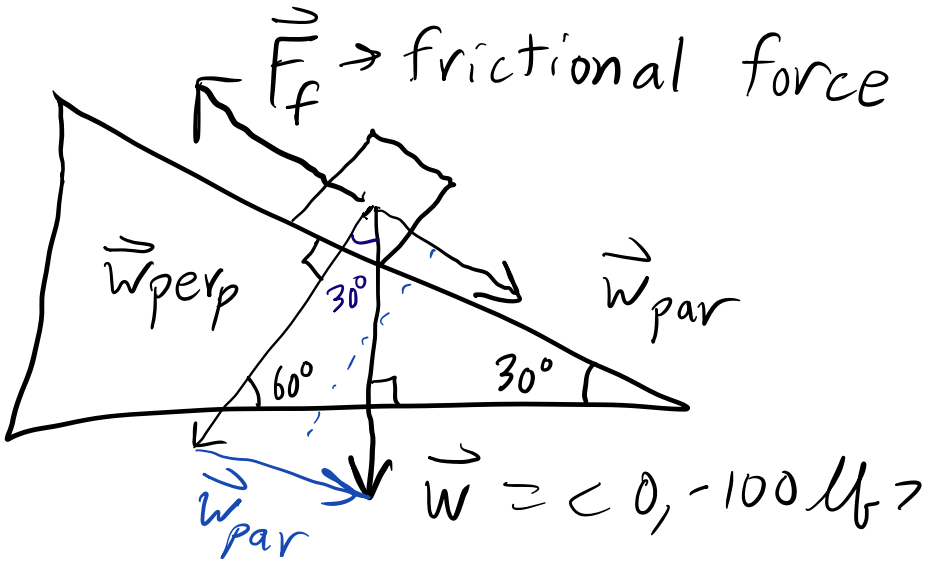
$$= \langle 25\sqrt{3}, -25\rangle$$

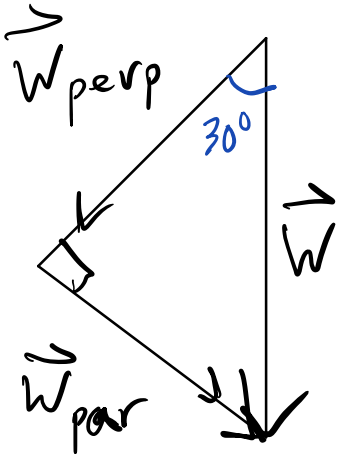
$$\vec{w}_{perp} = \vec{w} - \vec{w}_{par} = \langle 0, -100\rangle - \langle 25\sqrt{3}, -25\rangle$$

$$= \langle -25\sqrt{3}, -75\rangle$$

$$|\vec{w}_{par}| = |\vec{F}_f| = \mu |\vec{w}_{perp}| \rightarrow$$

$$\mu = \frac{|\vec{w}_{par}|}{|\vec{w}_{perp}|} = \frac{50}{50\sqrt{3}} = \frac{1}{\sqrt{3}} \quad (= \tan 30^\circ)$$





$$\mu = \frac{|\vec{w}_{par}|}{|\vec{w}_{perp}|} = \tan(30°)$$

$$= \frac{1}{\sqrt{3}}$$