Problem 14.1.4.45 Match function a-f with the appropriate graph A-F.

line a. $\vec{r}(t) = \langle t, -t, t\rangle. = \vec{0} + t\langle 1,-1,1\rangle$

b. $\vec{r}(t) = \langle t^2, t, t\rangle.$ $x = y^2,\ y = z$

c. $\vec{r}(t) = \langle 4\cos(t), 4\sin(t), 2\rangle.$

$x^2+y^2=16$

d. $\vec{r}(t) = \langle 2t, \sin(t), \cos(t)\rangle.$ $y^2+z^2=1$

e. $\vec{r}(t) = \langle \sin(t), \cos(t), \sin(2t)\rangle.$

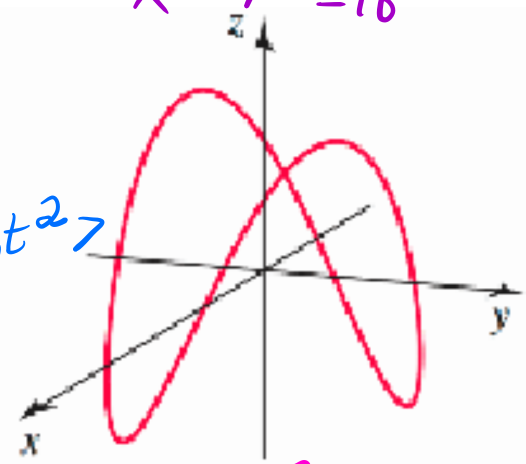
$x^2+y^2=1$

f. $\vec{r}(t) = \langle \sin(t), 2t, \cos(t)\rangle.$

$x^2+z^2=1$

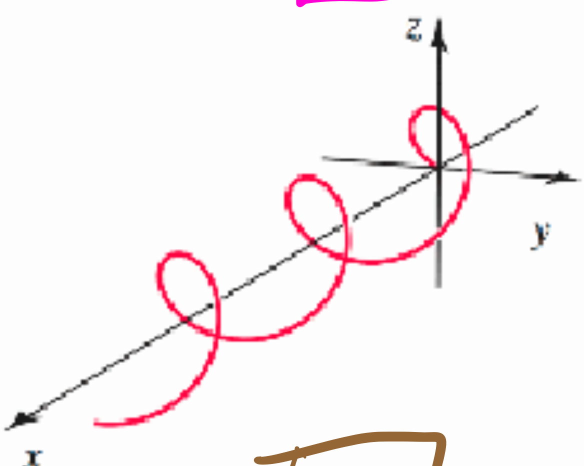
$y = x^2 \rightarrow$

$\vec{r}(t) = \langle t, t^2\rangle$

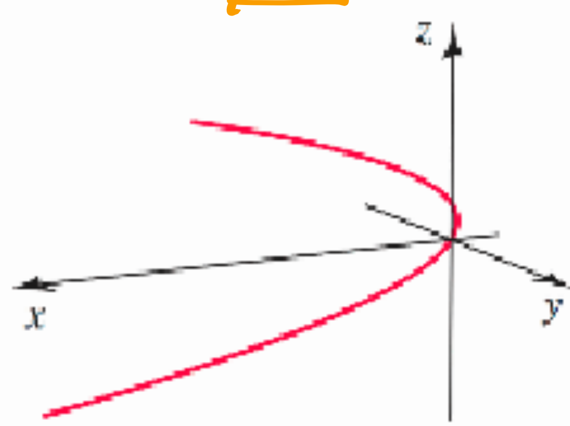


(A)

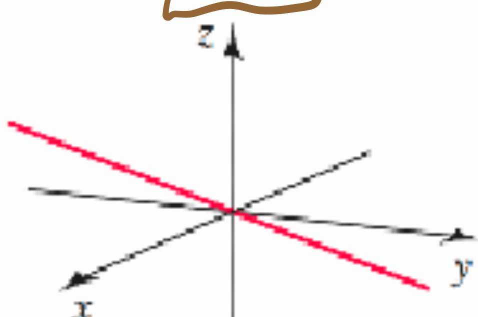
(B)



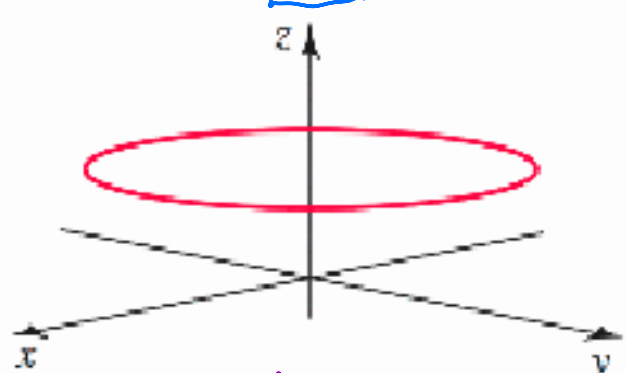
(C)



(D)



(E)

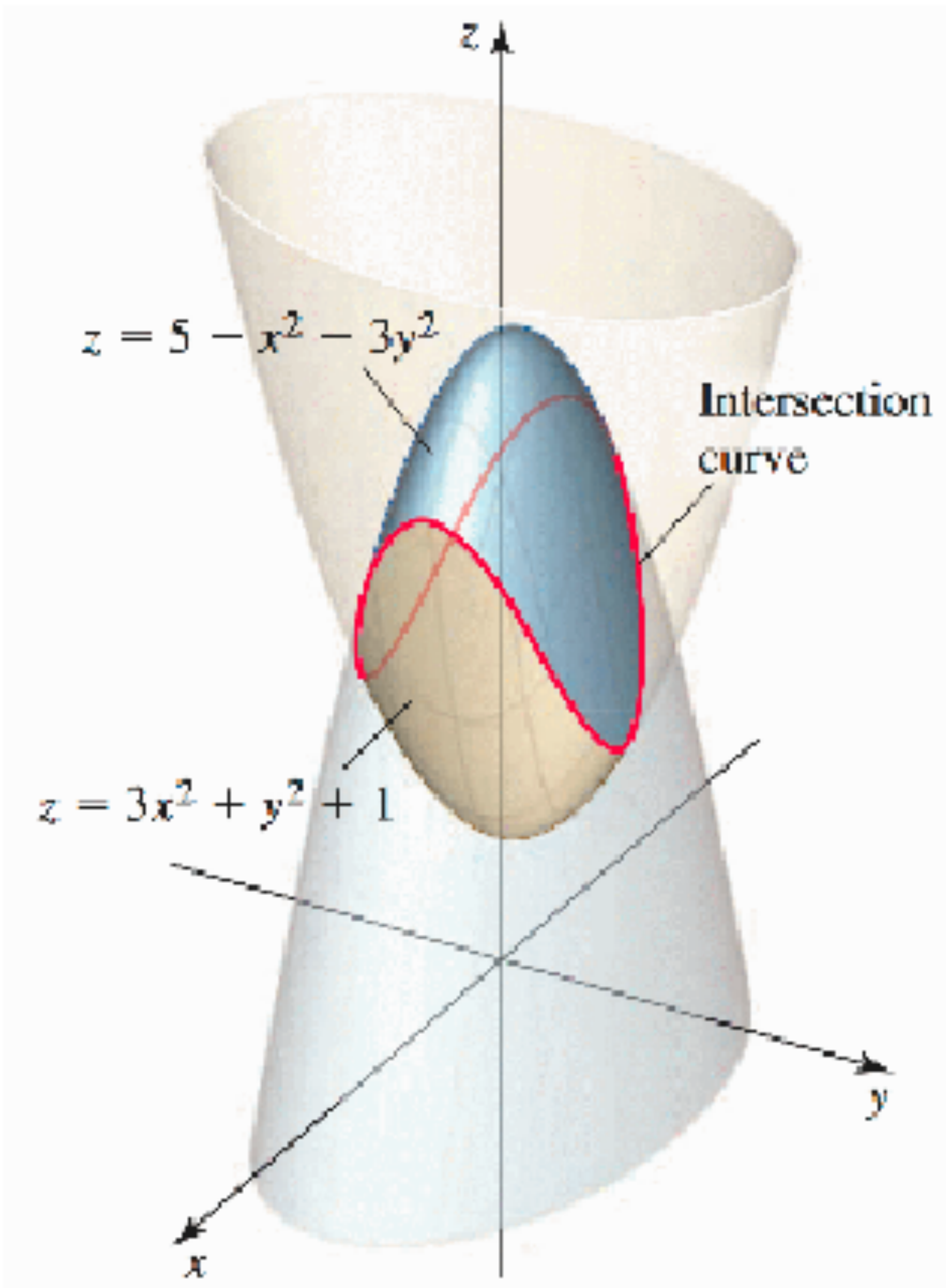


(F)

$-1 \le \sin(t) \le 1$ and $-1 \le \cos(t) \le 1$

Problem 14.1.4.48: Find a function $\vec{r}(t)$ that describes the curve which is the intersection of the surfaces $z = 3x^2 + y^2 + 1$ and $z = 5 - x^2 - 3y^2$. Note that there is not a unique answer to this question since any curve possess infinitely many distinct paramterizations.

$z = 3x^2 + y^2 + 1; z = 5 - x^2 - 3y^2$



$z = 5 - x^2 - 3y^2,\ z = 3x^2 + y^2 + 1 \rightarrow$

$5 - x^2 - 3y^2 = 3x^2 + y^2 + 1 \rightarrow$

$4 = 4x^2 + 4y^2 \rightarrow x^2 + y^2 = 1 \rightarrow$

$\vec{r}(t) = \langle \cos(t), \sin(t), ??? \rangle$

$0 \leq t \leq 2\pi$

$z(t) = 3x(t)^2 + y(t)^2 + 1$

$= 3\cos^2(t) + \sin^2(t) + 1$

$\cos^2(t) + \sin^2(t) = 1$

$= 2\cos^2(t) + 2 \rightarrow$

$$\boxed{\begin{gathered}\vec{r}(t) = \langle \cos(t), \sin(t), 2\cos^2(t) + 2 \rangle \\ 0 \leq t \leq 2\pi\end{gathered}}$$

$2\cos^2(t) + 2 = \cos(2t) + 3$

Problem 14.2.3.39-42: Suppose that $\vec{u}(t)$ and $\vec{v}(t)$ are differentiable vector valued functions satisfying $\vec{u}(0) = \langle 0,1,1\rangle, \vec{u}\,'(0) = \langle 0,7,1\rangle, \vec{v}(0) = \langle 0,1,1\rangle$, and $\vec{v}\,'(0) = \langle 1,1,2\rangle$. Evaluate the following expressions.

$\boxed{39.}$ $\frac{d}{dt}(\vec{u}(t)\cdot\vec{v}(t))\Big|_{t=0}$

$\boxed{40.}$ $\frac{d}{dt}(\vec{u}(t)\times\vec{v}(t))\Big|_{t=0}$

$\boxed{41.}$ $\frac{d}{dt}(\cos(t)\vec{u}(t))\Big|_{t=0}$

$\boxed{42.}$ $\frac{d}{dt}(\vec{u}(\sin(t)))\Big|_{t=0}$

$$\frac{d}{dt}(\vec{u}(t)\cdot\vec{v}(t)) = \vec{u}\,'(t)\cdot\vec{v}(t)+\vec{u}(t)\cdot\vec{v}\,'(t)$$

$$\frac{d}{dt}(\vec{u}(t)\cdot\vec{v}(t))\Big|_{t=0} = \vec{u}\,'(0)\cdot\vec{v}(0)+\vec{u}(0)\cdot\vec{v}\,'(0)$$

$$= \langle 0,7,1\rangle\cdot\langle 0,1,1\rangle + \langle 0,1,1\rangle\cdot\langle 1,1,2\rangle$$

$$= (0+7+1)+(0+1+2) = \boxed{11}$$

$$\frac{d}{dt}(\vec{u}(t)\times\vec{v}(t)) = \vec{u}\,'(t)\times\vec{v}(t)+\vec{u}(t)\times\vec{v}\,'(t)$$

$$\frac{d}{dt}(\vec{u}(t)\times\vec{v}(t))\Big|_{t=0} = \vec{u}\,'(0)\times\vec{v}(0)+\vec{u}(0)\times\vec{v}\,'(0)$$

$$= \langle 0,7,1\rangle\times\langle 0,1,1\rangle + \langle 0,1,1\rangle\times\langle 1,1,2\rangle$$

$$\boxed{= \ldots\ldots}$$

$$\frac{d}{dt}\left(f(t)\vec{u}(t)\right) = f'(t)\vec{u}(t) + f(t)\vec{u}'(t)$$

$$\frac{d}{dt}\left(\cos(t)\,\vec{u}(t)\right)\Big|_{t=0} = -\sin(0)\,\vec{u}(0) + \cos(0)\,\vec{u}'(0)$$

$$= 0\cdot\vec{u}(0) + 1\cdot\vec{u}'(0) = \vec{u}'(0) = \boxed{\langle 0,7,1\rangle}$$

$$\frac{d}{dt}\left(\vec{u}(f(t))\right) = \vec{u}'(f(t))\cdot f'(t)$$

$$= f'(t)\,\vec{u}'(f(t))$$

$$\frac{d}{dt}\left(\vec{u}(\sin(t))\right)\Big|_{t=0} = \cos(0)\,\vec{u}'(\sin(0))$$

$$= 1\cdot\vec{u}'(0) = \vec{u}'(0)$$

$$= \boxed{\langle 0,7,1\rangle}$$

Problem 14.2.3.27: Let $\vec{r}(t) = \langle t, 2, \frac{2}{t} \rangle$ for $t > 1$. Find the unit tangent vector $\hat{T}(t)$ at all points of the curve $\vec{r}(t)$.

$$\hat{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} \quad \left(\text{if } \vec{r}(t) \neq \vec{0}\right)$$

$$\vec{r}\,'(t) = \langle 1, 0, \frac{-2}{t^2} \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{1^2 + 0^2 + \left(\frac{-2}{t^2}\right)}$$

$$= \sqrt{1 + \frac{4}{t^4}} = \sqrt{\frac{t^4+4}{t^4}} = \frac{\sqrt{4+t^4}}{t^2}$$

$$\hat{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\langle 1, 0, \frac{-2}{t^2} \rangle}{\frac{\sqrt{4+t^4}}{t^2}}$$

$$= \frac{t^2}{\sqrt{4+t^4}} \langle 1, 0, \frac{-2}{t^2} \rangle$$

$t > 1$

$$= \left\langle \frac{t^2}{\sqrt{4+t^4}}, 0, \frac{-2}{\sqrt{4+t^4}} \right\rangle$$