Problem 1: Consider the function $f(x, y) = \ln(1 + 4x^2 + 3y^2)$ and the point $P = (\frac{3}{4}, -\sqrt{3})$.

a. Find the gradient field $\nabla f(x, y)$ of $f(x, y)$ and then evaluate it at P.

b. Find the angles θ (with respect to the x-axis) associated with the directions of maximum increase, maximum decrease, and zero change.

c. Write the directional derivative at P as a function of θ; call this function $g(\theta)$.

d. Find the value of θ that maximizes $g(\theta)$ and find the maximum value.

e. Verify that the value of θ that maximizes g corresponds to the direction of the gradient vector at P. Verify that the maximum value of g equals the magnitude of the gradient vector at P.

Problem 2: Consider the function $f(x, y) = x^2 + y^2$ and and the point $P = (2, 3)$.

(a) Find the unit vector that points in direction of maximum decrease of the function f at the point P.

(b) Calculate the directional derivative of f at the point P in the direction of the vector $\vec{u} = \langle 3, 2 \rangle$.

Problem 3: Below is a contour plot of some function $z = f(x, y)$ along with 4 vectors.

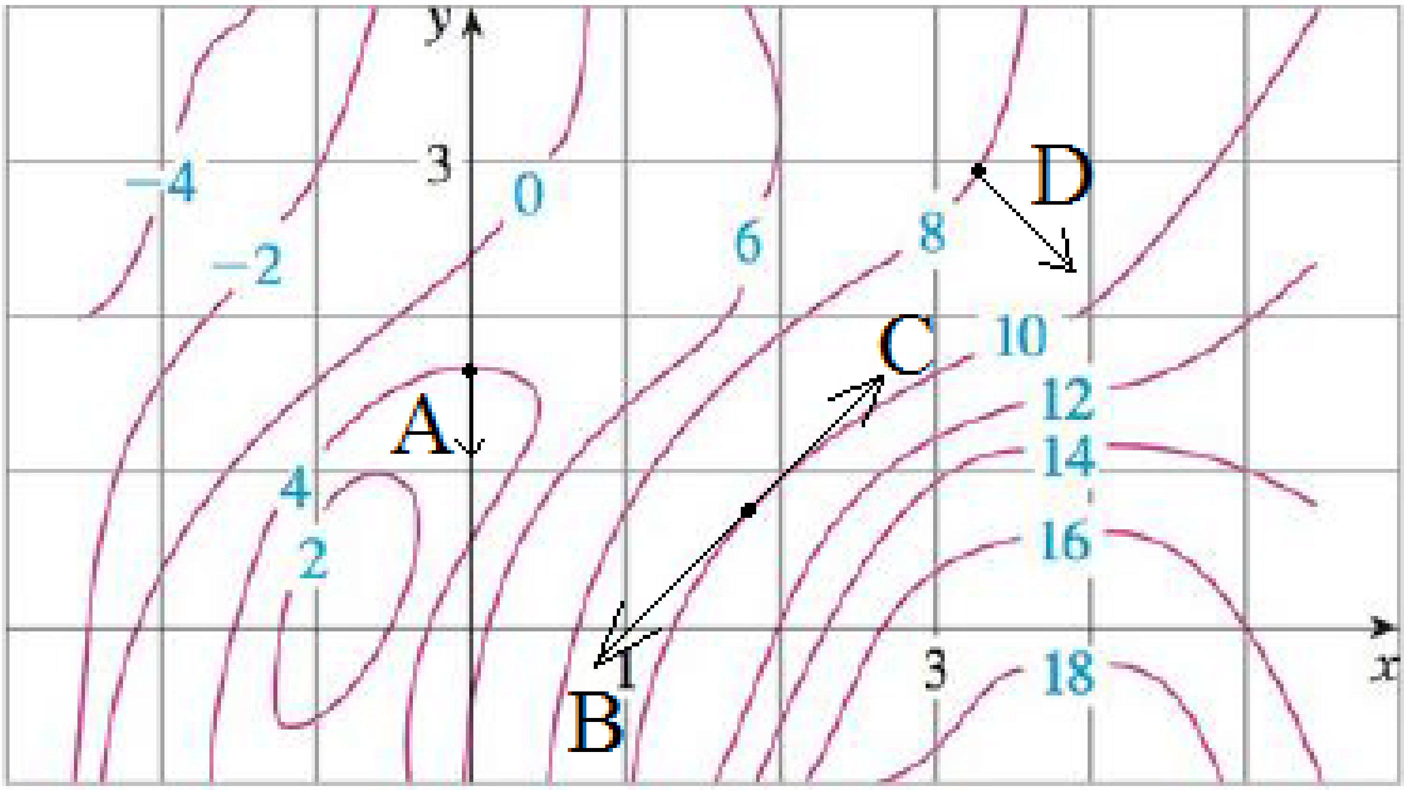

FIGURE 1. Contour plot of $z = f(x, y)$.

Which of the vectors in the above plot could possibly be a gradient vector of the function $f(x, y)$? Please circle all that apply.

(A) (B) (C) (D)
None of the vectors could possibly be a gradient vector for $f(x, y)$.

Problem 4: Determine all critical points of the function $f(x, y) = x^3 - y^3 + xy$, then classify each of the critical points as a local maximum, local minimum, or saddle point.

Problem 5: Show that the second derivative test is inconclusive when applied to the function $f(x,y) = x^4y^2$ at the point $(0,0)$. Show that $f(x,y)$ has a local minimum at $(0,0)$ by direct analysis.
Hint: The product of 2 negative numbers is positive.

Problem 6: Consider the function $f(x, y) = 3+x^4+3y^4$. Show that $(0, 0)$ is a critical point for $f(x, y)$ and show that the second derivative test is inconclusive at $(0, 0)$. Then describe the behavior of $f(x, y)$ at $(0, 0)$.