Problem 15.2.5.82: Verify that

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x)+\sin(y)}{x+y} = 1. \tag{1}$$

Homemade Problem: Consider the function

$$f(x,y) = \frac{xy^2}{x^2+y^4}. \tag{2}$$

(a) Show that if L is a line that passes through the origin, then

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in L}} f(x,y) = 0. \tag{3}$$

(b) Show that

$$\lim_{(x,y)\to(0,0)} f(x,y) \tag{4}$$

does not exist.

Problem 15.3.97: Consider the function $f(x, y) = \sqrt{|xy|}$.

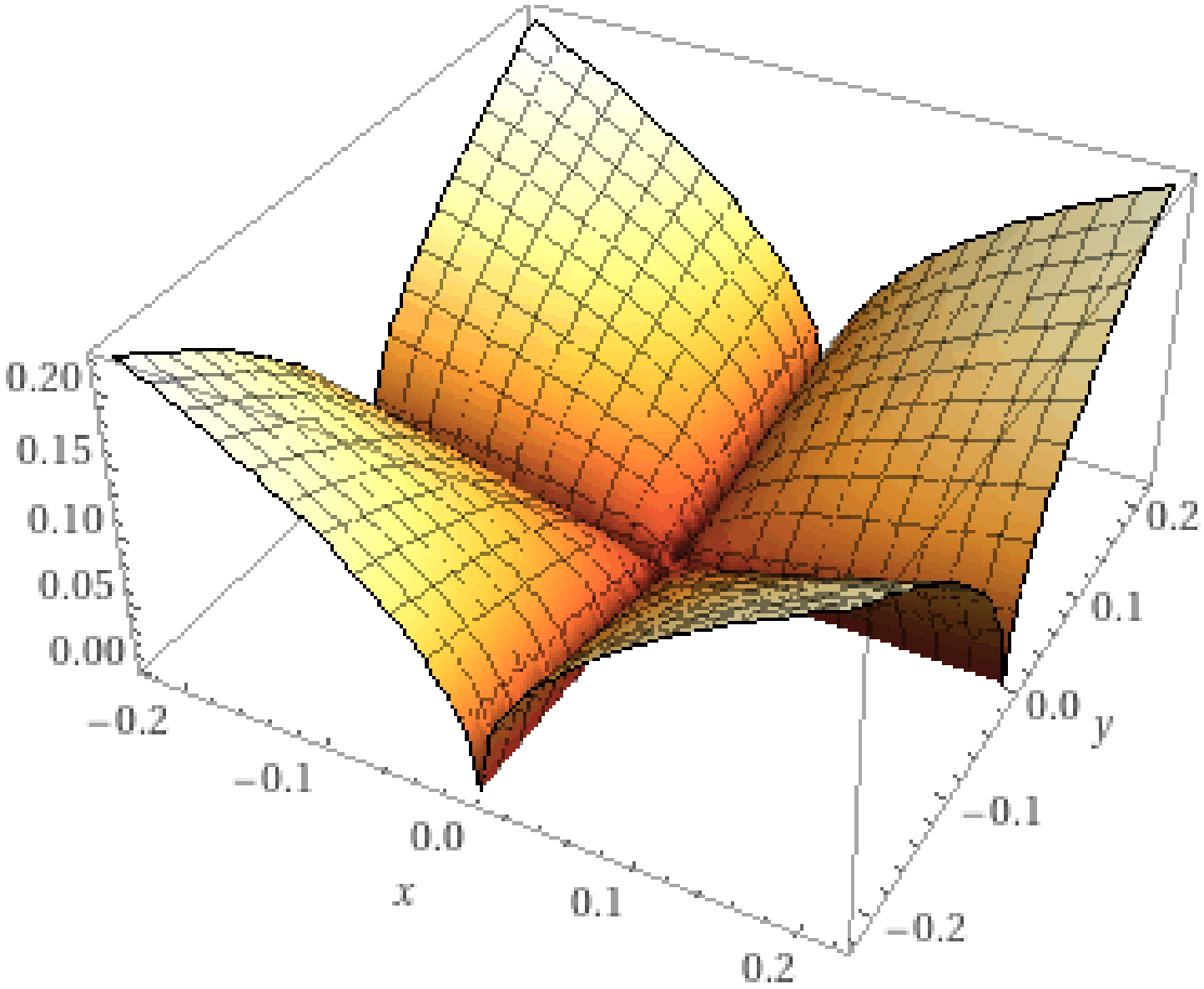

FIGURE 1. A graph of $z = \sqrt{|xy|}$.

(a) Is f continuous at $(0, 0)$?
(b) Show that $f_x(0, 0)$ and $f_y(0, 0)$ exist by calculating their values.
(c) Determine whether f_x and f_y are continuous at $(0, 0)$.
(d) Is f differentiable at $(0, 0)$?