vector $\vec{v} = \langle a, b \rangle$

point (a, b)

$\frac{\vec{v}}{\|\vec{v}\|}$ (if $\vec{v} \neq \vec{0}$, $\|\vec{0}\| = 0$)

$\|\vec{v}\|$ is the length of $\vec{v}$. $\|\vec{v}\| = \sqrt{a^2+b^2}$.

(comes from the Pythagorean theorem)

$\left(\begin{array}{l} \vec{0} = \langle 0, 0 \rangle \\ \vec{0} = \langle 0, 0, 0 \rangle \end{array} \right)$

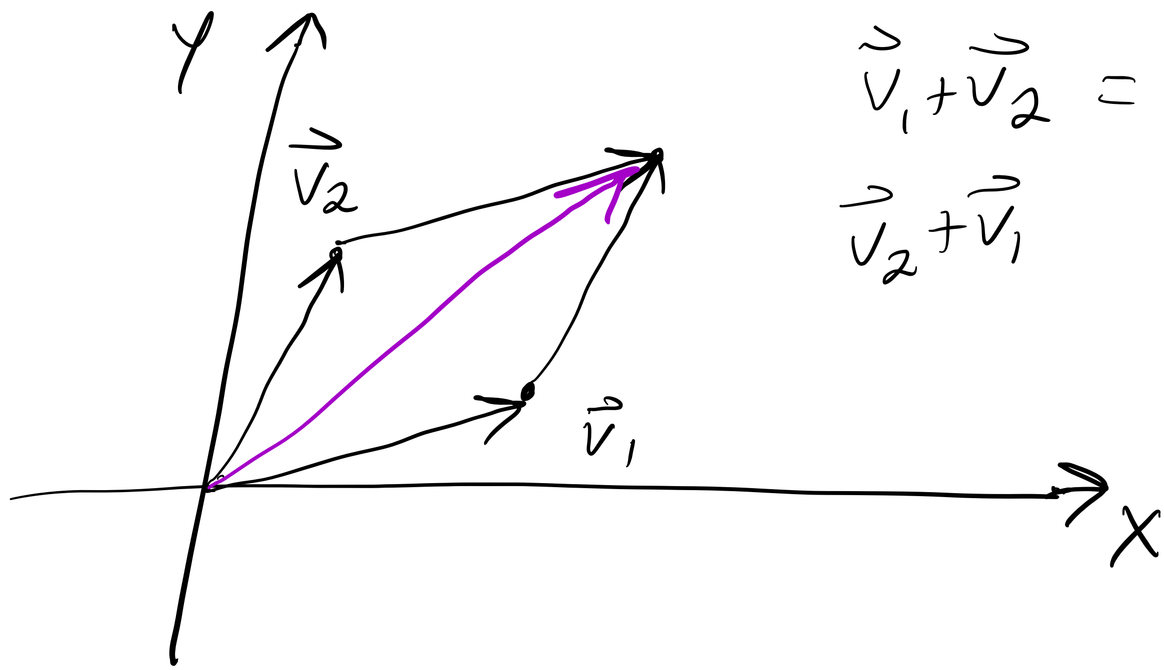
$\frac{\vec{v}}{\|\vec{v}\|}$ is the normalization of $\vec{v}$.

$\left\| \frac{\vec{v}}{\|\vec{v}\|} \right\| = 1$, and $\frac{\vec{v}}{\|\vec{v}\|}$ points in the same direction as $\vec{v}$.

Vector arithmetic

$\vec{v}_1 = \langle a_1, b_1 \rangle, \vec{v}_2 = \langle a_2, b_2 \rangle$

- $\boxed{\vec{v}_1 + \vec{v}_2} = \langle a_1 + a_2, b_1 + b_2 \rangle$



$\vec{v}_1 + \vec{v}_2 =$

$\vec{v}_2 + \vec{v}_1$

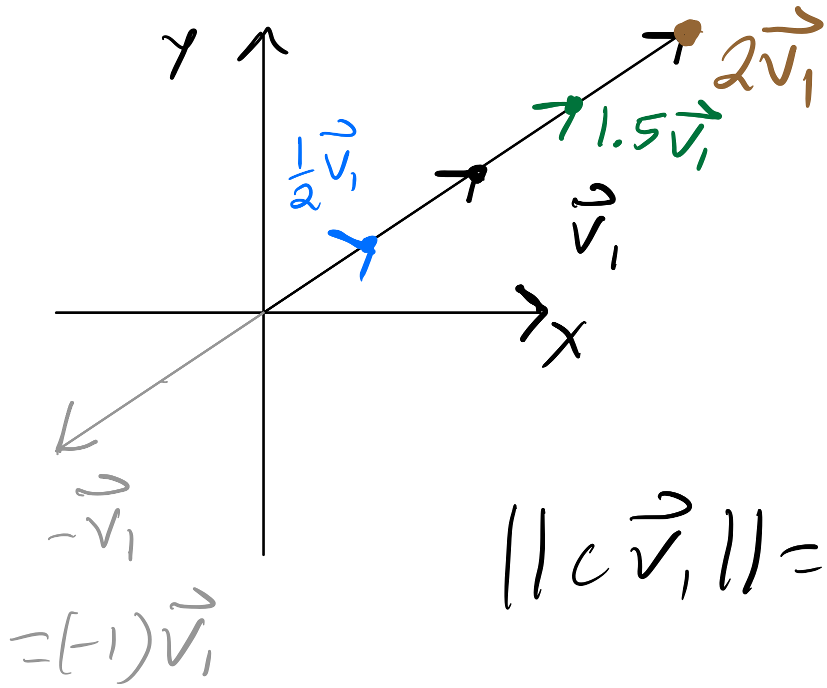
real numbers

- Given a scalar $c \in \mathbb{R}$,

$$c\vec{v}_1 = \langle ca_1, cb_1 \rangle.$$

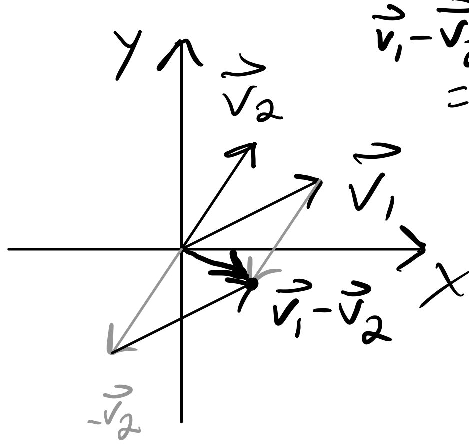
if $\vec{v}_1 = \langle a_1, b_1, c_1 \rangle$, then

$$c\vec{v}_1 = \langle ca_1, cb_1, c\,c_1 \rangle$$



$$\|c\vec{v}_1\| = |c| \cdot \|\vec{v}_1\|$$

- vector subtraction



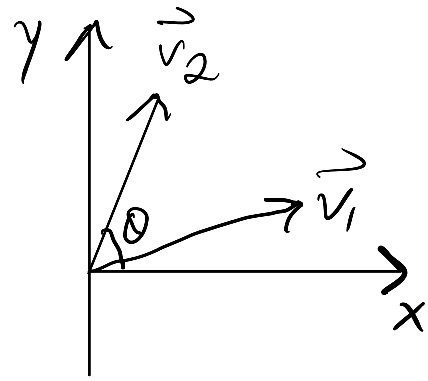
$\vec{v}_1-\vec{v}_2$??
$= (-1)\vec{v}_2+\vec{v}_1$

$\vec{v}_1-\vec{v}_2 = \langle a_1-a_2, b_1-b_2 \rangle$

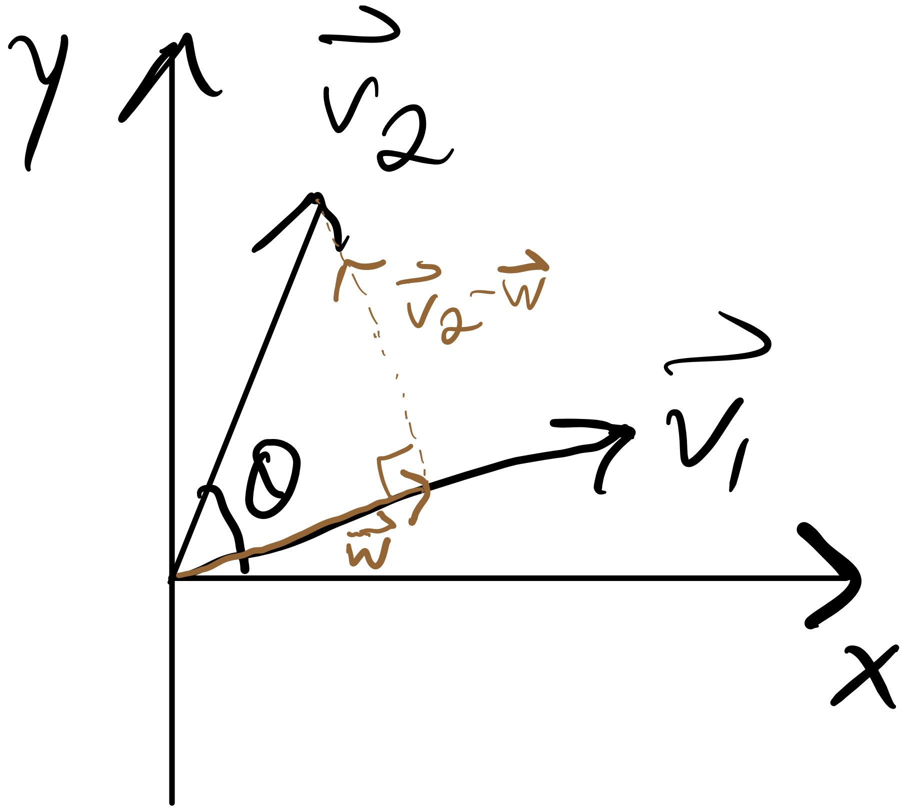
Dot products

Given $\vec{v}_1 = \langle a_1, b_1 \rangle$, $\vec{v}_2 = \langle a_2, b_2 \rangle$,

$$\underbrace{\vec{v}_1 \cdot}_{\text{dot product}} \vec{v}_2 = a_1 a_2 + b_1 b_2$$



$$\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cos(\theta)$$



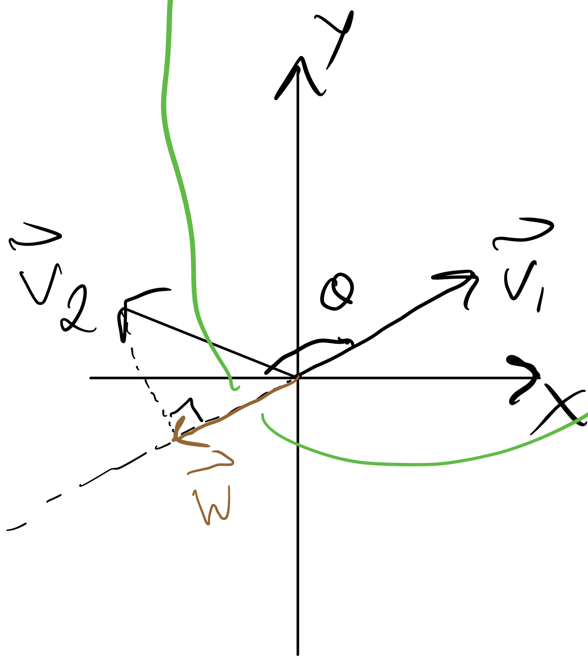
$\vec{w}$ is the projection of $\vec{v}_2$ onto $\vec{v}_1$.

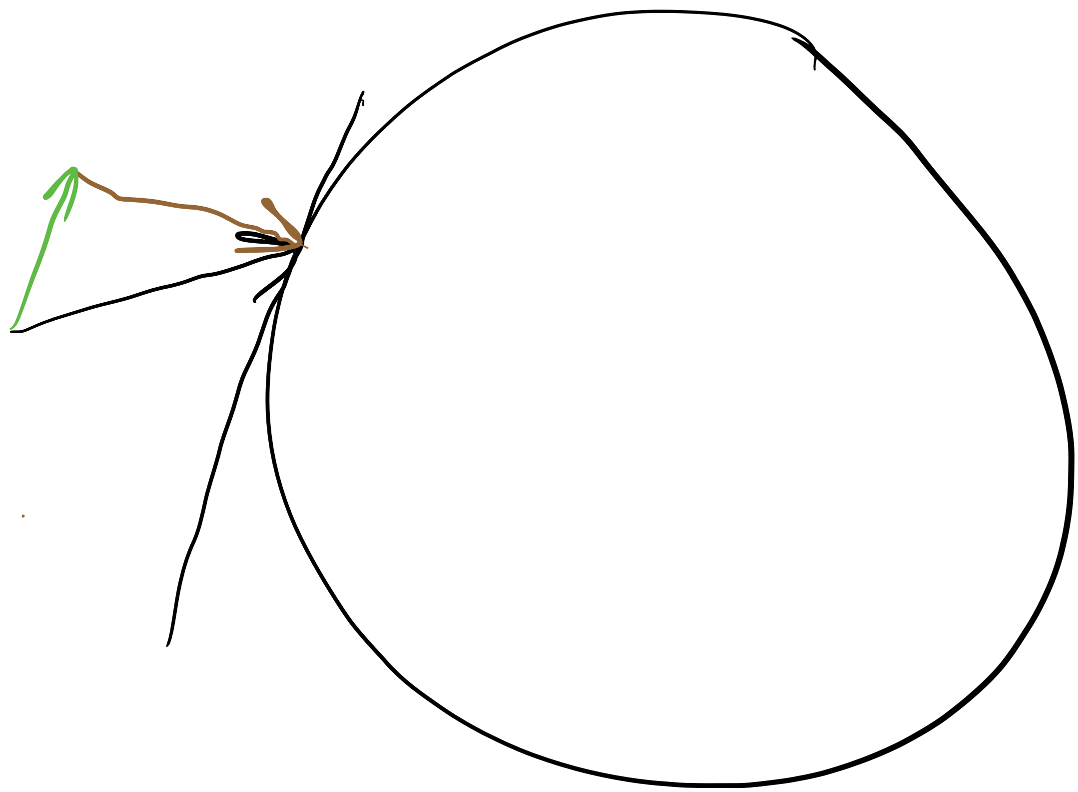
$$\frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\|} = \|\vec{v}_2\| \cos(\theta) = \|\vec{w}\|$$

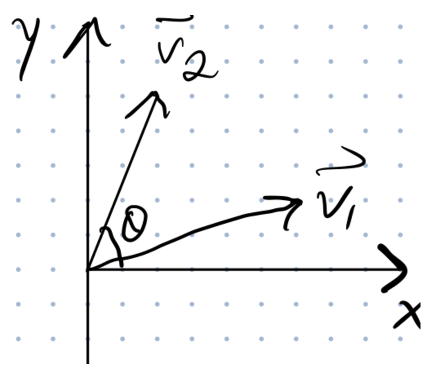
$\dfrac{\vec{v}_1}{\|\vec{v}_1\|}$ "is the direction of $\vec{v}_1$"

$$\rightarrow \vec{w} = \underbrace{\frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\|}}_{\|\vec{w}\|} \cdot \frac{\vec{v}_1}{\|\vec{v}_1\|} = \boxed{\frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\|^2}\vec{v}_1}$$

$$\|\vec{w}\| \cdot \frac{\vec{w}}{\|\vec{w}\|} = \frac{\vec{v}_1}{\|\vec{v}_1\|}$$







y
$\vec{v}_2$
θ
$\vec{v}_1$
x