4.3.8) A homogeneous system of equations with more variables than equations has infinitely many solutions. linear

4.3.14) A system of equations with more variables than equations will either have infinitely many solutions or no solutions. Since the given system has at least 1 solution, it must have infinitely many.

4.3.16) It will either have the unique solution

$(x_1, x_2, x_3) = (0,0,0)$, or it will have infinitely many solutions.

(Because the system is homogeneous)

4.3.17) A homogeneous system of linear equations either has the unique solution given by setting all variables equal to 0, or it has infinitely many solutions. In this case $(x_1, x_2) = (0,0)$ or $(1,-1)$ are both solutions, so the system has infinitely many solutions.

4.3.19) See 4.3.8 above.

4.3.21) $x_3 = 0 \Rightarrow x_2 = 0 \Rightarrow x_1 = 0$

4.3.22) $x_1 = 0 \Rightarrow x_2 = 0.$

4.5.6) $2A + 5B + D = 2B + 3A \Rightarrow D = (2B+3A) - (2A+5B)$

$$= 2B - 5B + 3A - 2A = -3B + A$$

$$= -3\begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ -3 & -9 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 2 & 4 \\ -2 & -6 \end{bmatrix}}$$

4.5.5) $A+2B+2D=3B \Rightarrow 2D=(3B)-(A+2B)$

$= 3B-2B-A = B-A \Rightarrow$

$D=\frac{1}{2}(B-A)$

$=\frac{1}{2}\left(\begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}-\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$=\frac{1}{2}\left(\begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix}\right)$

$=\boxed{\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}}$

4.5.17) $2\vec{r}+\vec{t} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix}+\begin{bmatrix} 1 \\ 4 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}+\begin{bmatrix} 1 \\ 4 \end{bmatrix}=\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$a_1\vec{s}+a_2\vec{u} = a_1\begin{bmatrix} 2 \\ -3 \end{bmatrix}+a_2\begin{bmatrix} -4 \\ 6 \end{bmatrix}=\begin{bmatrix} 2a_1-4a_2 \\ -3a_1+6a_2 \end{bmatrix}$

$a_1\vec{s}+a_2\vec{u}=2\vec{r}+\vec{t} \Rightarrow \begin{bmatrix} 2a_1-4a_2 \\ -3a_1+6a_2 \end{bmatrix}=\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ so.

$2a_1-4a_2=3$
$-3a_1+6a_2=4$
$\rightarrow \begin{array}{ccc} \left[\begin{array}{cc|c} 2 & -4 & 3 \\ -3 & 6 & 4 \end{array}\right] \\ \vec{s} \quad \vec{u} \quad 2\vec{r}+\vec{t} \end{array}$

$R_2+\frac{3}{2}R_1$: $\left[\begin{array}{cc|c} 2 & -4 & 3 \\ 0 & 0 & 4+\frac{9}{2} \end{array}\right] \rightarrow$ $2a_1-4a_2=3$

$\boxed{0=\frac{17}{2}} \rightarrow \boxed{\text{no solutions}}$

4.5.23) $w_1 = C\vec{r} = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} -2+0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$w_2 = Bw_1 = \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 \\ -2+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$w_3 = Aw_2 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+1 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$BC = \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+-1 & 0+-1 \\ -2+3 & 3+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 \\ 1 & 6 \end{bmatrix}$

$A(BC) = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} -2+1 & -2+6 \\ -1+3 & -1+18 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 \\ 2 & 17 \end{bmatrix}$

$(A(BC))\vec{r} = \begin{bmatrix} -1 & 4 \\ 2 & 17 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ ✓

4.5.37)

$$CA = \begin{bmatrix} 2 & 1 \\ 4 & 0 \\ 8 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4+1 & 6+4 \\ 8+0 & 12+0 \\ 16-1 & 24-4 \\ 6+2 & 9+8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 8 & 12 \\ 15 & 20 \\ 8 & 17 \end{bmatrix}$$