2.7.42) Evaluate $\iiint_D x\,dV$, where D is the region bounded by the planes $y-2x=0$, $y-2x=1$, $z-3y=0$, $z-3y=1$, $z-4x=0$ and $z-4x=3$.

Let $u=y-2x$, $v=z-3y$, and $w=z-4x$, so $0\le u\le 1$, $0\le v\le 1$, $0\le w\le 3$. Now we have to solve for x,y,z in terms of u,v,w.

$$u=-2x+y$$

$$v=-3y+z \rightarrow 3u+v=-6x+3y-3y+z=-6x+z$$

$$w=-4x+z \qquad 3u+v-w=-6x+z-(-4x)-z=-2x, \text{ so}$$

$$x=\frac{w}{2}-\frac{v}{2}-\frac{3}{2}u \qquad J(u,v,w)=\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$y=u+2x=w-v-2u$$

$$z=w+4x=3w-2v-6u,$$

$$=\begin{vmatrix} -\frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ -2 & -1 & 1 \\ -6 & -2 & 3 \end{vmatrix}=\left(-\frac{3}{2}\right)\cdot\begin{vmatrix} -1 & 1 \\ -2 & 3 \end{vmatrix}-\left(-\frac{1}{2}\right)\begin{vmatrix} -2 & 1 \\ -6 & 3 \end{vmatrix}$$

$$+\left(\frac{1}{2}\right)\cdot\begin{vmatrix} -2 & -1 \\ -6 & -2 \end{vmatrix}$$

$$=-\frac{3}{2}(-3-(-2))+\frac{1}{2}(-6-(-6))+\frac{1}{2}(4-6)=\frac{3}{2}-1=\frac{1}{2}.$$

$$\iiint_D x\,dV = \iiint_E \left(\frac{w}{2}-\frac{v}{2}-\frac{3}{2}u\right)|J(u,v,w)|\,dV$$

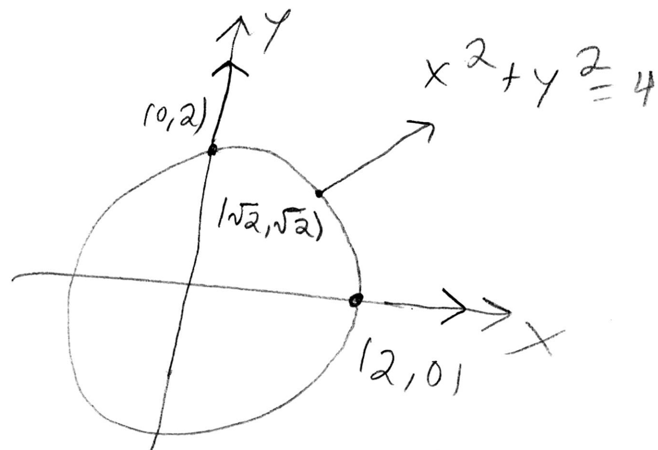
$$=\int_0^1\int_0^1\int_0^3 \left(\frac{w}{4}-\frac{v}{4}-\frac{3u}{4}\right)dw\,dv\,du = \int_0^1\int_0^1 \frac{w^2}{8}-\frac{vw}{4}-\frac{3uw}{4}\Big|_{w=0}^{3} dv\,du$$

$$=\int_0^1\int_0^1\left(\frac{9}{8}-\frac{3v}{4}-\frac{9u}{4}\right)dv\,du = \int_0^1 \frac{9v}{8}-\frac{3v^2}{8}-\frac{9uv}{4}\Big|_{v=0}^{1} du$$

$$=\int_0^1\left(\frac{9}{8}-\frac{3}{8}-\frac{9u}{4}\right)du=\int_0^1\left(\frac{3}{4}-\frac{9u}{4}\right)du=\frac{3u}{4}-\frac{9u^2}{8}\Big|_{u=0}^{1}$$

$$=\frac{3}{4}-\frac{9}{8}=\boxed{-\frac{3}{8}}$$

3.1.17)



$\vec{F}(c(t))\cdot c'(t) =$

$(\cos(t),\sin(t))\cdot(-\sin(t),\cos(t))$

$= -\sin(t)\cos(t)+\sin(t)\cos(t) = 0$,

for all $0\le t\le 2\pi$, so $\vec{F}$ is orthogonal to the entire curve.

$(x(t),y(t)) = c(t) = (\cos(t),\sin(t))$

$0\le t\le 2\pi$

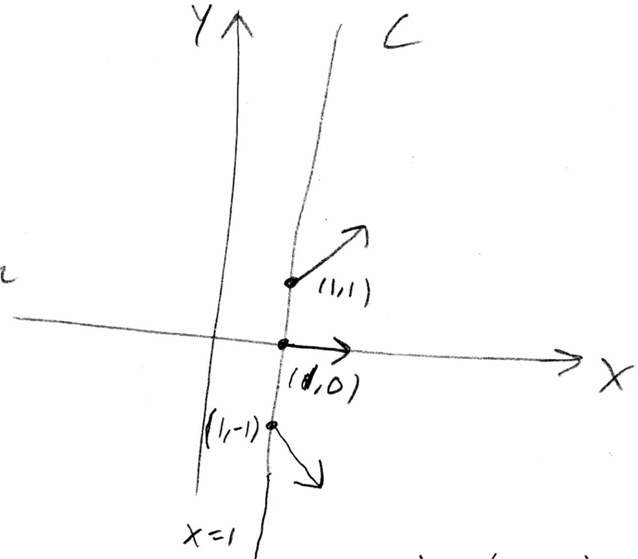
$c'(t) = (-\sin(t),\cos(t))$

$\vec{F}(x(t),y(t)) = (x(t),y(t)) = (\cos(t),\sin(t))$

19) $\vec{F}(c(t)) = \vec{F}(1,t) = (1,t)$.

$\vec{F}(c(t)) \cdot c'(t) = (1,t)\cdot(0,1) = t$.

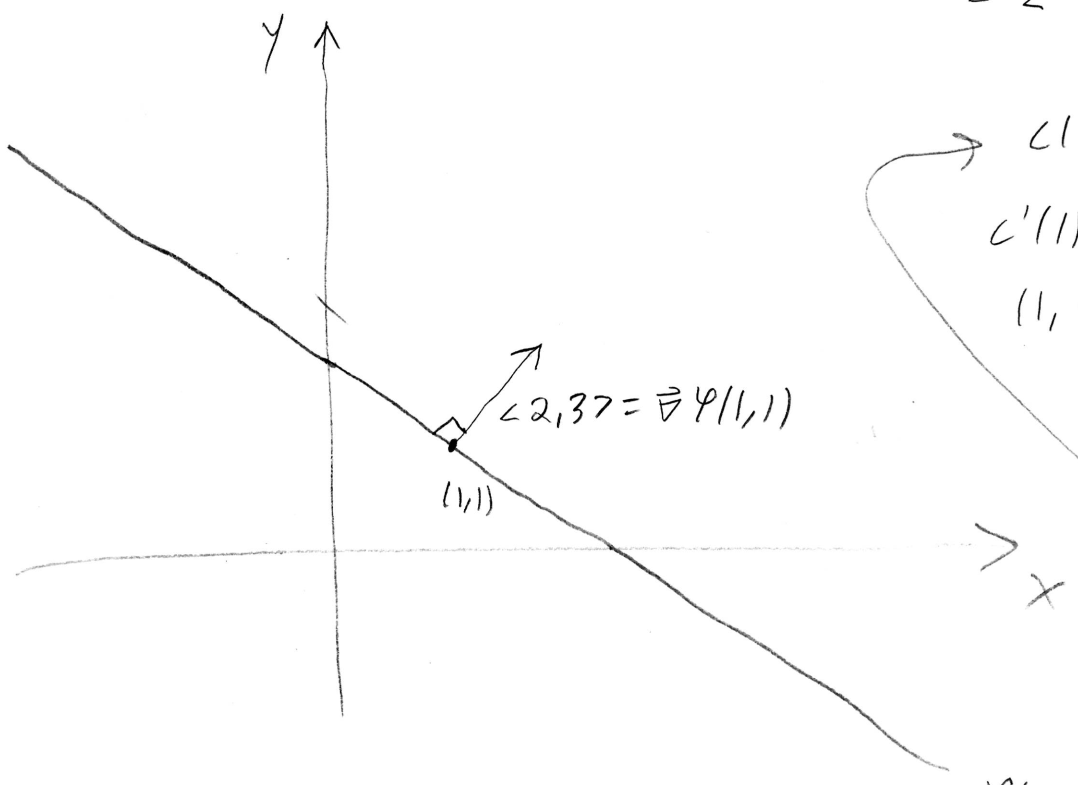
$\vec{F}$ is orthogonal to c only when $t=0$. $\vec{F}$ is never a multiple of $c'(t)$, so it is never tangent to $c(t)$.



$c(t) = (1,t)$,
$c'(t) = (0,1)$.

Let the class do 3.1.18 and 3.1.20 as groupwork if we have time. Then 3.1.16 as well.

3.1.37) $\varphi(x,y) = 2x+3y$. $\vec{\nabla}\varphi(x,y) = \langle \varphi_x(x,y), \varphi_y(x,y)\rangle = \langle 2,3\rangle$.



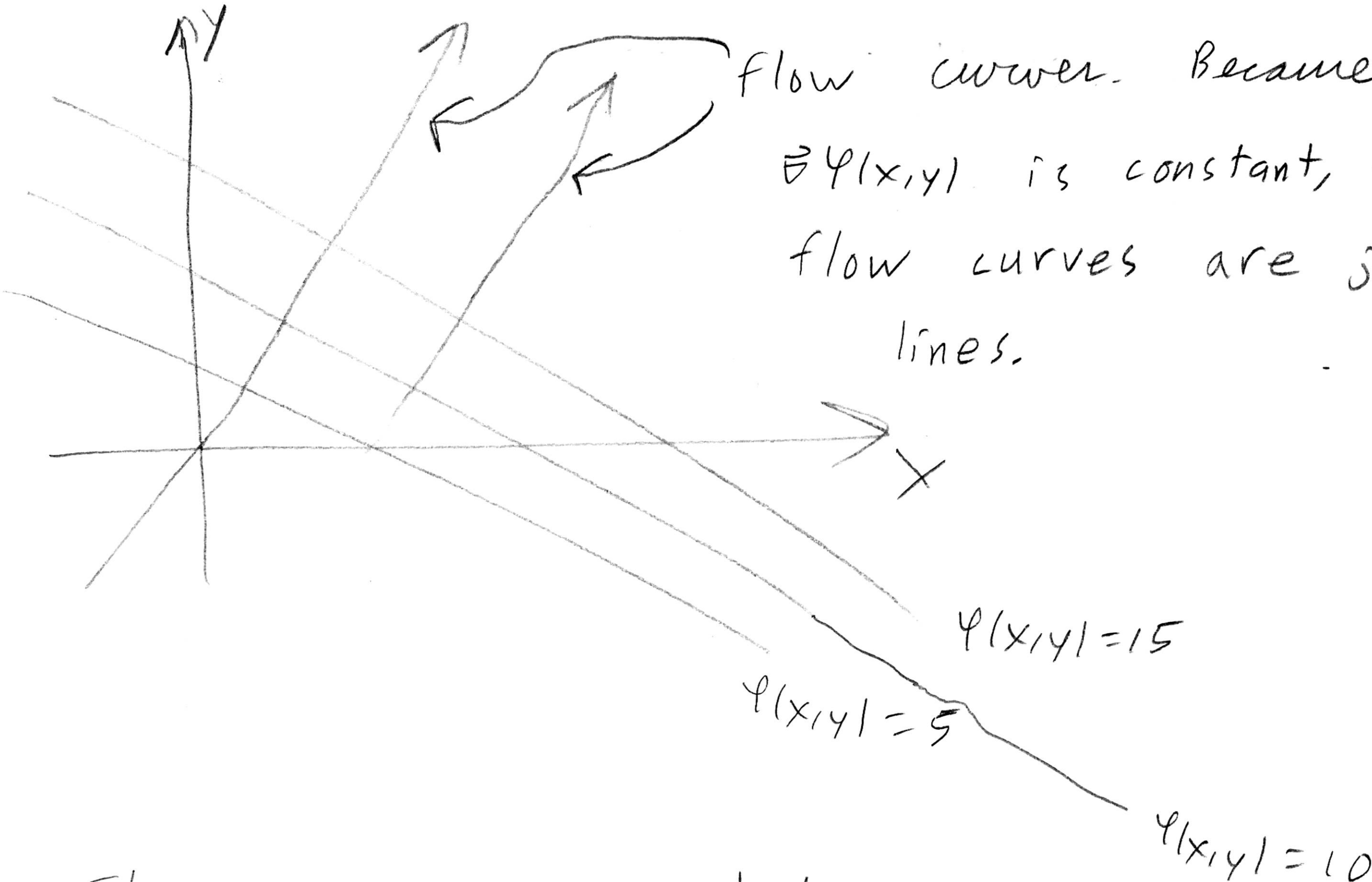
$c(1) = (1,1)$,

$c'(1)\cdot\vec{\nabla}\varphi(1,1) = (1,-\frac{2}{3})\cdot(2,3) = 2+(-\frac{2}{3})\cdot 3 = 0$, so $\vec{\nabla}\varphi(1,1)$ is orthogonal to $\varphi(x,y)=5$ at $(1,1)$.

$\varphi(x,y)=5 \Rightarrow 2x+3y=5 \Rightarrow y = \frac{5-2x}{3}$,

so $c(t) = (t, \frac{5-2t}{3}) \rightarrow c'(t) = (1,-\frac{2}{3})$.

Flow curves. Because $\vec{\nabla}\varphi(x,y)$ is constant, the flow curves are just lines.



- Flow curves are what happens when a particle is "pushed" by the vector field $\vec{\nabla}\varphi(x,y)$ and we track its path.

- If d is a constant, then the level curve $\varphi(x,y)=d$ can be parameterized as $c(t)=(t, \frac{d-2t}{3})$, so $\vec{c}\,'(t)=\langle 1, \frac{-2}{3}\rangle$, so $\vec{\nabla}\varphi(c(t))\cdot\vec{c}\,'(t)=\langle 2,3\rangle\cdot\langle 1,-\frac{2}{3}\rangle=0$, hence $\vec{\nabla}\varphi(c(t))$ is orthogonal to the level curve through $c(t)$.