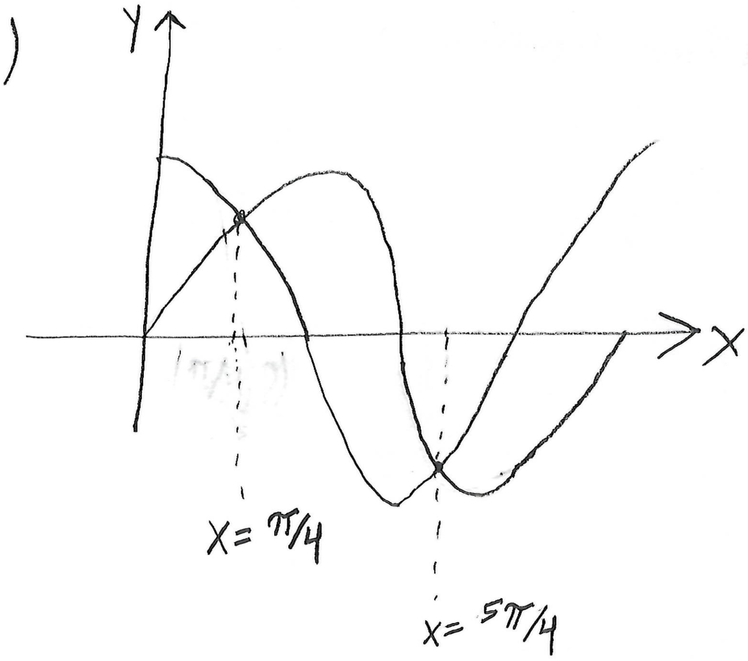
2.2.9+ Bonus)



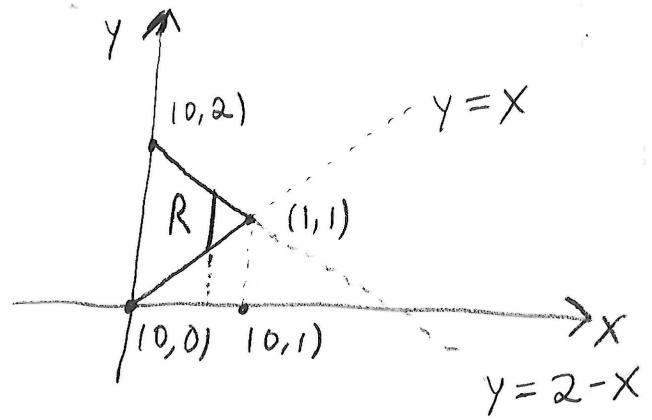
$$\iint_R f(x,y)\,dy\,dx = \int_0^{\pi/4}\int_{\sin(x)}^{\cos(x)} f(x,y)\,dy\,dx$$

Consider the region R_2 bound between $x=0$, $x=\frac{5\pi}{4}$, $y=\sin(x)$ and $y=\cos(x)$.

$$\iint_R f(x,y)\,dy\,dx = \int_0^{\pi/4}\int_{\sin(x)}^{\cos(x)} f(x,y)\,dy\,dx + \int_{\pi/4}^{5\pi/4}\int_{\cos(x)}^{\sin(x)} f(x,y)\,dy\,dx.$$

This is why it is important to graph the regions.

14)



Note: You should be able to find the equation of a line through 2 points.

$$\iint_R f(x,y)\,dA = \int_0^1 \int_{2-x}^{x} f(x,y)\,dA$$

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22) $$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2x^2y\,dy\,dx = \int_0^1 x^2y^2 \Big|_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} dx$$

$$= \int_0^1 \left(x^2\left(\sqrt{1-x^2}\right)^2 - x^2\left(-\sqrt{1-x^2}\right)^2\right) dx = \int_0^1 0\,dx = 0.$$

2.2.31)

$y=3x-9$ intersects the x-axis at $(3,0)$.
If we use $dA=dy\,dx$, then

$$\int_0^3 \int_0^{2x} f(x,y)\,dy\,dx + \int_3^9 \int_{3x-9}^{2x} f(x,y)\,dy\,dx.$$

$y=2x$
$\downarrow$
$x=y/2$

$y=3x-9$
$\downarrow$
$x=\frac{y+9}{3}$

If we use $dA=dx\,dy$, then

$$\int_0^{18} \int_{y/2}^{\frac{y+9}{3}} f(x,y)\,dx\,dy.$$

2.2.58) $y = 6 - 2x \Rightarrow x = \frac{6-y}{2}$

$$\int_0^3 \int_0^{6-2x} f(x,y)\,d\ d \quad = \quad \int_0^6 \int_0^{\frac{6-y}{2}} f(x,y)\,dx\,dy$$